

# Chapter 28

## List of Problems

## A Few Words

We gather some of the problems arising at various places in the main text. They are listed in order of occurrence and no suggestion is made about the degree of their difficulty. In fact, I have not put the same amount of effort in tackling all problems in the list. However, solving them, regardless of difficulty, will definitively inform the theory and the context in which they appear.

Each problem consists of a **task**, which is, in my estimation, the most likely outcome. The **alternative** is also explicitly given. When a task seems a bit involved and I am less certain about its status, I inserted in the list some special subtasks that might be easier to tackle. So, not all problems appearing in the list are independent, and, when aware, I tried to state this explicitly.

As for the format, here is a generic sample, before presenting the list:

**Problem X** (Chapter 00, Example 00)

This is a special case of Problem Y.

**Task:** Construct a  $\pi$ -institution in class A but not in class B.

**Alternative:** Prove that every  $\pi$ -institution in A is also in B.

## The List

**Problem 1** (Chapter 11, Example 811)

**Task:** Construct a  $\pi$ -institution which is syntactically prealgebraic, but not syntactically protoalgebraic.

**Alternative:** Prove that every syntactically prealgebraic  $\pi$ -institution is also syntactically protoalgebraic.

**Problem 2** (Chapter 11, Example 820)

**Task:** Construct a Suszko truth equational  $\pi$ -institution which is not Leibniz truth equational.

**Alternative:** Prove that every Suszko truth equational  $\pi$ -institution is also Leibniz truth equational.

Proving the conjecture (Task) would show that the truth equationality hierarchy of Chapter 11, Section 11.6 consists of two different classes. Proving the Alternative would reduce the hierarchy to a single class.

**Problem 3** (Chapter 11, Example 872)

**Task:** Construct a  $\pi$ -institution which is left truth equational, but not (family) truth equational.

**Alternative:** Prove that every left truth equational  $\pi$ -institution is also (family) truth equational.

**Problem 4** (Chapter 11, Example 890)

**Task:** Construct a  $\pi$ -institution which is system truth equational, but not left truth equational.

**Alternative:** Prove that every system truth equational  $\pi$ -institution is also left truth equational.

**Problem 5** (Chapter 12, Example 936)

**Task:** Construct a  $\pi$ -institution which is syntactically weakly algebraizable, but not syntactically weakly family algebraizable.

**Alternative:** Prove that every syntactically weakly algebraizable  $\pi$ -institution is also syntactically weakly family algebraizable.

**Problem 6** (Chapter 12, Example 955)

**Task:** Construct a  $\pi$ -institution which is syntactically weakly left c-reflective prealgebraizable, but not syntactically weakly algebraizable.

**Alternative:** Prove that every syntactically weakly left c-reflective prealgebraizable  $\pi$ -institution is also syntactically weakly algebraizable.

**Problem 7** (Chapter 12, Example 956)

**Task:** Construct a  $\pi$ -institution which is syntactically weakly system prealgebraizable, but not syntactically weakly left c-reflective prealgebraizable.

**Alternative:** Prove that every syntactically weakly system prealgebraizable  $\pi$ -institution is also syntactically weakly left c-reflective prealgebraizable.

**Problem 8** (Chapter 13, Example 990)

**Task:** Construct a  $\pi$ -institution which is syntactically preequivalential, but not syntactically equivalential.

**Alternative:** Prove that every syntactically preequivalential  $\pi$ -institution is also syntactically equivalential.

**Problem 9** (Chapter 13, Example 1021)

**Task:** Construct a  $\pi$ -institution which is strongly left truth equational, but not strongly (family) truth equational.

**Alternative:** Prove that every strongly left truth equational  $\pi$ -institution is also strongly (family) truth equational.

**Problem 10** (Chapter 13, Example 1037)

**Task:** Construct a  $\pi$ -institution which is strongly system truth equational, but not strongly left truth equational.

**Alternative:** Prove that every strongly system truth equational  $\pi$ -institution is also strongly left truth equational.

**Problem 11** (Chapter 13, Example 1045)

**Task:** Construct a  $\pi$ -institution which is syntactically left prealgebraizable

but not syntactically left antiprealgebraizable.

**Alternative:** Prove that every syntactically left prealgebraizable  $\pi$ -institution is also syntactically left antiprealgebraizable.

**Problem 12** (Chapter 13, Example 1046)

**Task:** Construct a  $\pi$ -institution which is syntactically left antiprealgebraizable but not syntactically left prealgebraizable.

**Alternative:** Prove that every syntactically left antiprealgebraizable  $\pi$ -institution is also syntactically left prealgebraizable.

Together with the preceding problem, this Task would establish that syntactic left prealgebraizability and syntactic left antiprealgebraizability are incomparable and that they are both strictly dominated by syntactic strong left prealgebraizability.

**Problem 13** (Chapter 13, Example 1061)

**Task:** Construct a  $\pi$ -institution which is syntactically system prealgebraizable but not syntactically system antiprealgebraizable.

**Alternative:** Prove that every syntactically system prealgebraizable  $\pi$ -institution is also syntactically system antiprealgebraizable.

**Problem 14** (Chapter 13, Example 1062)

**Task:** Construct a  $\pi$ -institution which is syntactically system antiprealgebraizable but not syntactically system prealgebraizable.

**Alternative:** Prove that every syntactically system antiprealgebraizable  $\pi$ -institution is also syntactically system prealgebraizable.

Together with the preceding problem, this Task would establish that syntactic system prealgebraizability and syntactic system antiprealgebraizability are incomparable and that they are both strictly dominated by syntactic strong system prealgebraizability.

**Problem 15** (Chapter 13, Example 1070)

**Task:** Construct a  $\pi$ -institution which is syntactically strongly system prealgebraizable but neither syntactically left prealgebraizable nor syntactically left antiprealgebraizable.

**Alternative:** Prove that every syntactically strongly system prealgebraizable  $\pi$ -institution is also syntactically left prealgebraizable and/or syntactically left antiprealgebraizable.

This Task would establish that the three classes of the syntactic left prealgebraizability subhierarchy are properly contained, one by one, in the corresponding classes of the syntactic system prealgebraizability subhierarchy.

**Problem 16** (Chapter 13, Example 1078)

**Task:** Construct a  $\pi$ -institution which is syntactically family algebraizable but not syntactically family antialgebraizable.

**Alternative:** Prove that every syntactically family algebraizable  $\pi$ -institution is also syntactically family antialgebraizable.

**Problem 17** (Chapter 13, Example 1079)

**Task:** Construct a  $\pi$ -institution which is syntactically family antialgebraizable but not syntactically family algebraizable.

**Alternative:** Prove that every syntactically family antialgebraizable  $\pi$ -institution is also syntactically family algebraizable.

Together with the preceding problem, this Task would establish that syntactic family algebraizability and syntactic family antialgebraizability are incomparable and that they are both strictly dominated by syntactic strong family algebraizability.

**Problem 18** (Chapter 13, Example 1094)

**Task:** Construct a  $\pi$ -institution which is syntactically algebraizable but not syntactically antialgebraizable.

**Alternative:** Prove that every syntactically algebraizable  $\pi$ -institution is also syntactically antialgebraizable.

**Problem 19** (Chapter 13, Example 1095)

**Task:** Construct a  $\pi$ -institution which is syntactically antialgebraizable but not syntactically algebraizable.

**Alternative:** Prove that every syntactically antialgebraizable  $\pi$ -institution is also syntactically algebraizable.

Together with the preceding problem, this Task would establish that syntactic algebraizability and syntactic antialgebraizability are incomparable and that they are both strictly dominated by syntactic strong algebraizability.

**Problem 20** (Chapter 13, Example 1103)

**Task:** Construct a  $\pi$ -institution which is syntactically strongly algebraizable but neither syntactically family algebraizable nor syntactically family antialgebraizable.

**Alternative:** Prove that every syntactically strongly algebraizable  $\pi$ -institution is also syntactically family algebraizable and/or syntactically family antialgebraizable.

This Task would establish that the three classes of the syntactic family algebraizability subhierarchy are properly contained, one by one, in the corresponding classes of the syntactic algebraizability subhierarchy.

**Problem 21** (Chapter 13, Example 1104)

**Task:** Construct a  $\pi$ -institution which is syntactically strongly left prealgebraizable but neither syntactically family algebraizable nor syntactically family antialgebraizable.

**Alternative:** Prove that every syntactically strongly left prealgebraizable  $\pi$ -institution is also syntactically family algebraizable and/or syntactically family antialgebraizable.

This Task would establish that the three classes of the syntactic algebraizability subhierarchy are properly contained, one by one, in the corresponding classes of the syntactic left prealgebraizability subhierarchy.

**Problem 22** (Chapter 14, Example 1198)

**Task:** Construct a  $\pi$ -institution which is roughly left truth equational, but not roughly/narrowly (family) truth equational.

**Alternative:** Prove that every roughly left truth equational  $\pi$ -institution is also roughly/narrowly (family) truth equational.

**Problem 23** (Chapter 14, Example 1199)

**Task:** Construct a  $\pi$ -institution which is roughly left truth equational, but not narrowly left truth equational.

**Alternative:** Prove that every roughly left truth equational  $\pi$ -institution is also narrowly left truth equational.

**Problem 24** (Chapter 14, Example 1200)

**Task:** Construct a  $\pi$ -institution which is narrowly left truth equational, but not roughly system truth equational.

**Alternative:** Prove that every narrowly left truth equational  $\pi$ -institution is also roughly system truth equational.

**Problem 25** (Chapter 14, Example 1201)

**Task:** Construct a  $\pi$ -institution which is roughly/narrowly (family) truth equational, but not roughly left truth equational.

**Alternative:** Prove that every roughly/narrowly (family) truth equational  $\pi$ -institution is also roughly left truth equational.

**Problem 26** (Chapter 14, Example 1202)

**Task:** Construct a  $\pi$ -institution which is roughly/narrowly (family) truth equational, but not narrowly left truth equational.

**Alternative:** Prove that every roughly/narrowly (family) truth equational  $\pi$ -institution is also narrowly left truth equational.

Together with the preceding four problems, this Task would clarify the status of the rough/narrow truth equationality hierarchy, as shown in the diagram at the end of Section 14.6.

**Problem 27** (Chapter 15, Theorem 1233)

**Task:** Give an intrinsic necessary and sufficient condition for an arbitrary narrowly family monotone  $\pi$ -institution to be syntactically narrowly family monotone.

**Problem 28** (Chapter 15, Theorem 1248)

**Task:** Give an intrinsic necessary and sufficient condition for an arbitrary narrowly system monotone  $\pi$ -institution to be syntactically narrowly system monotone.

**Problem 29** (Chapter 15, Theorem 1260)

**Task:** Give an intrinsic necessary and sufficient condition for an arbitrary narrowly right monotone  $\pi$ -institution to be syntactically narrowly right monotone.

**Problem 30** (Chapter 15, Example 1262)

**Task:** Construct a  $\pi$ -institution which is syntactically narrowly family monotone, but not syntactically narrowly right monotone.

**Alternative:** Prove that every syntactically narrowly family monotone  $\pi$ -institution is also syntactically narrowly right monotone.

**Problem 31** (Chapter 15, Example 1263)

**Task:** Construct a  $\pi$ -institution which is syntactically narrowly right monotone, but not syntactically narrowly family monotone.

**Alternative:** Prove that every syntactically narrowly right monotone  $\pi$ -institution is also syntactically narrowly family monotone.

Together with the preceding problem, this Task would clarify the status of the syntactic narrow monotonicity hierarchy, shown in the diagram at the end of Section 15.4.

**Problem 32** (Chapter 16, Example 1271)

**Task:** Construct a  $\pi$ -institution which is syntactically family regularly prealgebraic, but not syntactically system regularly protoalgebraic.

**Alternative:** Prove that every syntactically family regularly prealgebraic  $\pi$ -institution is also syntactically system regularly protoalgebraic.

**Problem 33** (Chapter 16, Example 1272)

**Task:** Construct a  $\pi$ -institution which is syntactically system regularly protoalgebraic, but not syntactically family regularly prealgebraic.

**Alternative:** Prove that every syntactically system regularly protoalgebraic  $\pi$ -institution is also syntactically family regularly prealgebraic.

Together with the preceding problem, this Task would clarify the status of the syntactic regular pre/protoalgebraicity hierarchy, shown in one of the diagrams in Section 16.3.

**Problem 34** (Chapter 16, Example 1278)

**Task:** Construct a  $\pi$ -institution which is syntactically family regularly pre-equivalential, but not syntactically system regularly equivalential.

**Alternative:** Prove that every syntactically family regularly pre-equivalen-

tial  $\pi$ -institution is also syntactically system regularly equivalential.

**Problem 35** (Chapter 16, Example 1279)

**Task:** Construct a  $\pi$ -institution which is syntactically system regularly equivalential, but not syntactically family regularly preequivalential.

**Alternative:** Prove that every syntactically system regularly equivalential  $\pi$ -institution is also syntactically family regularly preequivalential.

Together with the preceding problem, this Task would clarify the status of the syntactic regular (pre)equivalentiality hierarchy, shown in one of the diagrams in Section 16.4.

**Problem 36** (Chapter 16, Example 1286)

**Task:** Construct a  $\pi$ -institution which is syntactically left assertional, but not syntactically family assertional.

**Alternative:** Prove that every syntactically left assertional  $\pi$ -institution is also syntactically family assertional.

**Problem 37** (Chapter 16, Example 1287)

**Task:** Construct a  $\pi$ -institution which is syntactically system assertional, but not syntactically left assertional.

**Alternative:** Prove that every syntactically system assertional  $\pi$ -institution is also syntactically left assertional.

Together with the preceding problem, this Task would clarify the status of the syntactic assertional hierarchy, shown in one of the diagrams in Section 16.5.

**Problem 38** (Chapter 16, Example 1296)

**Task:** Construct a  $\pi$ -institution which is syntactically regularly weakly left prealgebraizable, but not syntactically regularly weakly family prealgebraizable.

**Alternative:** Prove that every syntactically regularly weakly left prealgebraizable  $\pi$ -institution is also syntactically regularly weakly family prealgebraizable.

**Problem 39** (Chapter 16, Example 1297)

**Task:** Construct a  $\pi$ -institution which is syntactically regularly weakly system prealgebraizable, but not syntactically regularly weakly left prealgebraizable.

**Alternative:** Prove that every syntactically regularly weakly system prealgebraizable  $\pi$ -institution is also syntactically regularly weakly left prealgebraizable.

Together with the preceding problem, this Task would clarify the status of the syntactic regular weak prealgebraizability hierarchy, shown in one of the diagrams in Section 16.6.

**Problem 40** (Chapter 16, Example 1299)

**Task:** Construct a  $\pi$ -institution which is syntactically family regularly protoalgebraic, but not syntactically regularly weakly system prealgebraizable.

**Alternative:** Prove that every syntactically family regularly protoalgebraic  $\pi$ -institution is also syntactically regularly weakly system prealgebraizable.

**Problem 41** (Chapter 16, Example 1301)

**Task:** Construct a  $\pi$ -institution which is syntactically family assertional, but not syntactically regularly weakly system prealgebraizable.

**Alternative:** Prove that every syntactically family assertional  $\pi$ -institution is also syntactically regularly weakly system prealgebraizable.

**Problem 42** (Chapter 16, Example 1303)

**Task:** Construct a  $\pi$ -institution which is syntactically weakly family algebraizable, but not syntactically regularly weakly system prealgebraizable.

**Alternative:** Prove that every syntactically weakly family algebraizable  $\pi$ -institution is also syntactically regularly weakly system prealgebraizable.

**Problem 43** (Chapter 16, Example 1314)

**Task:** Construct a  $\pi$ -institution which is syntactically regularly weakly system algebraizable, but not syntactically regularly weakly family algebraizable.

**Alternative:** Prove that every syntactically regularly weakly system algebraizable  $\pi$ -institution is also syntactically regularly weakly family algebraizable.

**Problem 44** (Chapter 16, Example 1315)

**Task:** Construct a  $\pi$ -institution which is syntactically regularly weakly left prealgebraizable, but not syntactically regularly weakly system algebraizable.

**Alternative:** Prove that every syntactically regularly weakly left prealgebraizable  $\pi$ -institution is also syntactically regularly weakly system algebraizable.

**Problem 45** (Chapter 16, Example 1326)

**Task:** Construct a  $\pi$ -institution which is syntactically regularly system algebraizable, but not syntactically regularly family algebraizable.

**Alternative:** Prove that every syntactically regularly system algebraizable  $\pi$ -institution is also syntactically regularly family algebraizable.

**Problem 46** (Chapter 16, Example 1327)

**Task:** Construct a  $\pi$ -institution which is syntactically regularly left prealgebraizable, but not syntactically regularly system algebraizable.

**Alternative:** Prove that every syntactically regularly left prealgebraizable  $\pi$ -institution is also syntactically regularly system algebraizable.

**Problem 47** (Chapter 16, Example 1328)

**Task:** Construct a  $\pi$ -institution which is syntactically regularly system prealgebraizable, but not syntactically regularly left prealgebraizable.

**Alternative:** Prove that every syntactically regularly system prealgebraizable  $\pi$ -institution is also syntactically regularly left prealgebraizable.

Together with the preceding two problems, this Task would clarify the status of the syntactic regular (pre)algebraizability hierarchy, shown in one of the diagrams in Section 16.8.

**Problem 48** (Chapter 16, Example 1330)

**Task:** Construct a  $\pi$ -institution which is syntactically family regularly equivalential, but not syntactically regularly system prealgebraizable.

**Alternative:** Prove that every syntactically family regularly equivalential  $\pi$ -institution is also syntactically regularly system prealgebraizable.

**Problem 49** (Chapter 16, Example 1332)

**Task:** Construct a  $\pi$ -institution which is syntactically regularly weakly family algebraizable, but not syntactically regularly system prealgebraizable.

**Alternative:** Prove that every syntactically regularly weakly family algebraizable  $\pi$ -institution is also syntactically regularly system prealgebraizable.

**Problem 50** (Chapter 16, Example 1334)

**Task:** Construct a  $\pi$ -institution which is syntactically family algebraizable, but not syntactically regularly system prealgebraizable.

**Alternative:** Prove that every syntactically family algebraizable  $\pi$ -institution is also syntactically regularly system prealgebraizable.