Federated $\mathcal{ALCI}$: Preliminary Report

George Voutsadakis, Giora Slutzki, Vasant Honavar
Department of Computer Science
Iowa State University
Ames, IA 50011

Jie Bao
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, NY 12180

***Abstract*— We introduce F-$\mathcal{ALCI}$, a federated version of the description logic $\mathcal{ALCI}$. An F-$\mathcal{ALCI}$ ontology, like its *package-based* counterpart $\mathcal{ALCIP}^-$, consists of multiple $\mathcal{ALCI}$ ontologies that can *import* concepts or roles defined in other modules. Unlike $\mathcal{ALCIP}^-$, which supports only contextualized negation, F-$\mathcal{ALCI}$, supports contextualization of each of the logical connectives, a feature that allows more flexible reuse of knowledge from independently developed ontologies. We provide a new semantics for F-$\mathcal{ALCI}$ based on image domain relations and establish the conditions that need to be imposed on domain relations to ensure properties, such as preservation of unsatisfiability and monotonicity of inference, that are desirable in distributed web applications. We also establish the *decidability* of F-$\mathcal{ALCI}$.**

I. Introduction

The semantic web , much like the world-wide web, relies on the *network effect*, that is on leveraging the work of independent actors who contribute resources, including ontologies, that are interlinked to form a web of resources. In short, *ontologies* : *semantic web* :: *web pages* : *web*. Inevitably, the axioms that make up such ontologies are applicable within the specific *contexts* that are implicitly assumed by their authors. Hence, semantic web applications that have to rely on knowledge from a network of interlinked ontologies need to carefully reconcile the disparate contexts assumed by different ontologies [4], [3].

Modular ontology formalisms, such as distributed description logics (DDL) $\mathcal{E}$-Connections, semantic importing, semantic binding and package-based description logics (P-DL) (see [2] and the references cited therein), provide support, to varying degrees, for exploiting a network of interlinked ontologies. With the exception of P-DL [1], [2] which provides limited support for explicit treatment of context through contextualized negation, none of the formalisms provides support for explicit reference to *context*.

Against this background, this paper explores a family of *contextualized federated description logics* (CFDLs) to explore the incorporation of context in a simplified, yet practically useful setting.

- Introduction of CFDL F-$\mathcal{ALCI}$, in which each of the individual ontology modules is expressed in the DL $\mathcal{ALCI}$ (the most fundamental description logic $\mathcal{ALC}$ augmented with inverse roles), which, to the best of our knowledge, represents the first modular ontology language to support contextualized interpretation of *all* logical connectives used within the DL modules. In the CFDL F-$\mathcal{ALCI}$, inferences are always drawn *from the point of view* of a *witness* module. The results of inference are guranteed to be the same as those obtained by a standard reasoner over an ontology that integrates (from the point of view of the witness module) the knowledge that it (selectively) imports from the other modules. Different modules might infer different consequences, based on the knowledge that they *import* from the other modules.
- Characterization of the tradeoffs between the restrictions on domain relations (and hence the semantics of the CFDL F-$\mathcal{ALCI}$) and the desirable features of the resulting modular ontologies (e.g., monotonicity of inference and transitive propagation of concept subsumptions across modules linked by importing relations). Specifically, we show that, in the general case, when interpretations with *arbitrary* domain relations are allowed in the semantics of F-$\mathcal{ALCI}$, several of the desirable properties, e.g., monotonicity of inference and preservation of unsatisfiability, are lost; and that regaining these properties requires strengthening the conditions on the semantics of F-$\mathcal{ALCI}$. Furthermore, we show that it is possible to preserve many of the desirable properties of P-DLs, while at the same time imposing milder restrictions than those used in P-DLs.

For the sake of simplicity, we have not considered cyclic importing or more expressive languages than $\mathcal{ALCI}$ (e.g., $\mathcal{SHOIQ}$). Nor have we dealt with A-Boxes or R-Boxes. Our experience with P-DLs suggests that incorporation of A-Boxes and R-Boxes are unlikely to present major challenges. However, the interplay between increased expressivity of language used by individual ontology modules, unrestricted importing relations (e.g., cyclic importing) and the semantics based on image domain relations requires further study.

II. The Federated Description Logic F-$\mathcal{ALCI}$

In [2], given an ordinary description logic $\mathcal{L}$, the notation $\mathcal{LP}$ is introduced to denote its package-based counterpart, i.e., the package-based description logic which uses $\mathcal{L}$ as the logical language in each of its packages. Furthermore, the notation $\mathcal{LP}^-$ signifies that the importing of concept names and role names across packages is acyclic. In the present work, we use the prefix "F-", standing for **F**ederated, to denote a contextualized federated language and, since our discussion is limited to acyclic importing, omit the use of a superscript "$^-$" from the notation.

In this section, the syntax and the semantics of the language F-$\mathcal{ALCI}$ will be described in some detail.

A. The Syntax

Suppose a directed acyclic graph $G = \langle V, E\rangle$, with $V = \{1, 2, \ldots, n\}$, is given. The intuition is that its n nodes correspond to local modules of a modular ontology and its edges correspond to the importing relations between these modules. For technical reasons, we add a loop on each vertex of G.

For every node $i \in V$, the signature of the i-language always includes a set $\mathcal{C}_i$ of i-**concept names** and a set $\mathcal{R}_i$ of i-**role names**. We assume that all sets of names are pairwise disjoint. Out of these, a set of *i-concept expressions* $\widehat{\mathcal{C}}_i$ and a set of *i-role expressions* $\widehat{\mathcal{R}}_i$ are built.

Recall that the description logic $\mathcal{ALCI}$ allows concept expressions that are constructed recursively from its signature symbols, i.e., its role and concept names, using negation, conjunction, disjunction, value and existential restriction and inverses of role names. Its formulas are subsumptions between concept expressions.

The syntax of the description logic F-$\mathcal{ALCI}$ is defined as follows:

Definition 1 (Roles and Concepts): The set of i-**roles** or i-**role expressions** $\widehat{\mathcal{R}}_i$ consists of expressions of the form R, R^-, with $R \in \mathcal{R}_j, (j, i) \in E$.

The set of i-**concepts** or i-**concept expressions** $\widehat{\mathcal{C}}_i$, on the other hand, is defined recursively as follows:

$$A \in \mathcal{C}_j, \top_j, \bot_j, \neg_j C, C \sqcap_j D, C \sqcup_j D, \exists_j R.C, \forall_j R.C, \quad (1)$$

where $(j, i) \in E$, $C, D \in \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j$ and $R \in \widehat{\mathcal{R}}_i \cap \widehat{\mathcal{R}}_j$.

Using the concepts and roles of F-$\mathcal{ALCI}$, we define its formulas, as follows:

Definition 2 (Formulas): The i-**formulas** are expressions of the form $C \sqsubseteq D$, with $C, D \in \widehat{\mathcal{C}}_i$, for all $i \in V$.

An F-$\mathcal{ALCI}$-**TBox** or **TBox** is a collection $T = \{T_i\}_{i \in V}$, where T_i is a finite set of i-formulas, called the i-**TBox**. Since, in this paper, we do not consider RBoxes or ABoxes, the terms *TBox*, *ontology* and *knowledge base* will be used interchangeably.

For all $i \in V$, $\overline{\mathcal{R}}_i$ and $\overline{\mathcal{C}}_i$ denote the set of i-roles and of i-concepts, respectively, that occur in T_i. $\overline{\mathcal{C}}_i$ is a finite subset of $\widehat{\mathcal{C}}_i$. A role name in $\mathcal{R}_j \cap \overline{\mathcal{R}}_i$ or a concept name in $\mathcal{C}_j \cap \overline{\mathcal{C}}_i$ is said to be **imported from** module j **to** module i. Furthermore, since $\overline{\mathcal{C}}_i \subseteq \widehat{\mathcal{C}}_i$, it is obvious that a module i is allowed to use logical connectives subscripted by the index of a module j, whenever $(j, i) \in E$.

B. The Semantics

In this subsection, we present the semantics for the language F-$\mathcal{ALCI}$.

Definition 3: An **interpretation** $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E}\rangle$ consists of a family $\mathcal{I}_i = \langle \Delta^i, \cdot^i\rangle, i \in V$, of **local interpretations**, together with a family of **image domain relations** $r_{ij} \subseteq \Delta^i \times \Delta^j, (i, j) \in E$, such that $r_{ii} = \mathrm{id}_{\Delta^i}$, for all $i \in V$.

Notation: For a binary relation $r \subseteq \Delta^i \times \Delta^j$, $X \subseteq \Delta^i$ and $S \subseteq \Delta^i \times \Delta^i$, we set

$$r(X) := \{y \in \Delta^j : (\exists x \in X)((x, y) \in r)\},$$

$$r(S) := \{(z, w) \in \Delta^j \times \Delta^j : (\exists (x, y) \in S)((x, z), (y, w) \in r)\}.$$

A local interpretation function $\cdot^i$ interprets i-role names and i-concept names, as well as $\bot_i$ and $\top_i$, as follows:

- $C^i \subseteq \Delta^i$, for all $C \in \mathcal{C}_i$,
- $R^i \subseteq \Delta^i \times \Delta^i$, for all $R \in \mathcal{R}_i$,
- $\top_i^i = \Delta^i$, $\bot_i^i = \emptyset$.

The interpretations of imported role names and imported concept names are computed by the following rules:

- $C^i = r_{ji}(C^j)$, for all $C \in \mathcal{C}_j \cap \widehat{\mathcal{C}}_i$,
- $R^i = r_{ji}(R^j)$, for all $R \in \mathcal{R}_j \cap \widehat{\mathcal{R}}_i$,
- $\top_j^i = r_{ji}(\Delta^j)$, $\bot_j^i = \emptyset$.

The recursive features of the local interpretation function $\cdot^i$ are as follows:

- $R^{-i} = R^{i-}$, for all $R \in \mathcal{R}_i$,
- $(\neg_j C)^i = r_{ji}(\Delta^j - C^j)$
- $(C \sqcap_j D)^i = r_{ji}(C^j \cap D^j)$
- $(C \sqcup_j D)^i = r_{ji}(C^j \cup D^j)$
- $(\exists_j R.C)^i = r_{ji}(\{x \in \Delta^j : (\exists y)((x, y) \in R^j$ and $y \in C^j)\})$
- $(\forall_j R.C)^i = r_{ji}(\{x \in \Delta^j : (\forall y)((x, y) \in R^j$ implies $y \in C^j)\})$

For all $i \in V$, i-**satisfiability**, denoted by $\models^i$, is defined by $\mathcal{I} \models^i C \sqsubseteq D$ iff $C^i \subseteq D^i$. Given a TBox $T = \{T_i\}_{i \in V}$, the interpretation $\mathcal{I}$ is a **model** of T_i, written $\mathcal{I} \models^i T_i$, iff $\mathcal{I} \models^i \tau$, for every $\tau \in T_i$. Moreover, $\mathcal{I}$ is a **model** of T, written $\mathcal{I} \models T$, iff $\mathcal{I} \models^i T_i$, for every $i \in V$.

Let $w \in V$. Define $G_w = \langle V_w, E_w\rangle$ to be the subgraph of G induced by those vertices in G from which w is reachable and $T_w^* := \{T_i\}_{i \in V_w}$. We say that an F-$\mathcal{ALCI}$-ontology $T = \{T_i\}_{i \in V}$ is **consistent as witnessed by a module** T_w if T_w^* has a model $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i \in V_w}, \{r_{ij}\}_{(i,j) \in E_w}\rangle$, such that $\Delta^w \neq \emptyset$. A concept C is **satisfiable as witnessed by** T_w if there is a model $\mathcal{I}$ of T_w^*, such that $C^w \neq \emptyset$. A concept subsumption $C \sqsubseteq D$ **is valid as witnessed by** T_w, denoted by $C \sqsubseteq_w D$, if, for every model $\mathcal{I}$ of T_w^*, $C^w \subseteq D^w$. An alternative notation for $C \sqsubseteq_w D$ is $T_w^* \models_w C \sqsubseteq D$.

Example 1. (See Figure 1). This example illustrates contextualized intersection and contextualized negation in F-$\mathcal{ALCI}$. Consider a module P_i, containing concepts "Computer Model" and "Phone Model". Module P_j, on the other hand, has a concept "Brand". Suppose module P_j imports concepts "Computer model" and "Phone Model" from module P_i. An interpretation of the ontology is given in Figure 1. The image domain relation r_{ij} maps both a computer model "MacBook Air" and a phone model "iPhone" to the brand "Apple", an instance of the concept "Brand" in P_j. One may verify that, whereas $(\text{Computer Model} \sqcap_i \text{Phone Model})^j = \emptyset$, we have that $(\text{Computer Model} \sqcap_j \text{Phone Model})^j \neq \emptyset$. Intuitively, $(\text{Computer Model} \sqcap_j \text{Phone Model})^j$ contains brands in domain Δ^j that are associated with both individuals of "Computer Model" and individuals of "Phone Model" in module P_i. The brand "Apple" fulfills this requirement.

One may also verify that, in this case, $(\neg_i \text{Computer Model})^j = \text{Brand}^j$ (individuals in domain

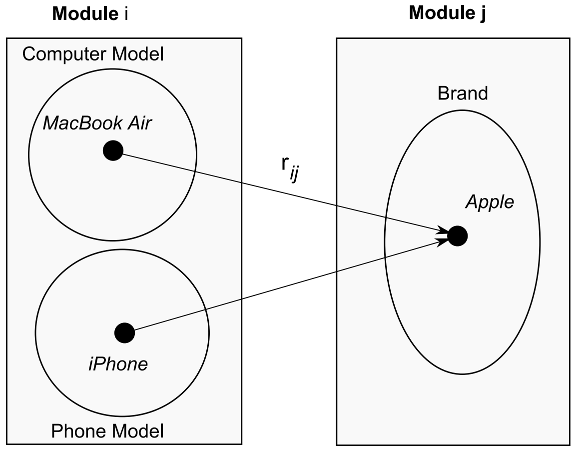

Fig. 1. Interpretation of Contextualized Negation.

Δ^j that are associated with individuals that are not computer models in domain Δ^i), whereas $(\neg_j \text{Computer Model})^j = \emptyset$ (individuals in domain Δ^j that are not associated with individuals that are computer models in domain Δ^i). This example shows that negation in F-$\mathcal{ALCI}$ has contextualized meaning in different modules.

III. Exactness of F-$\mathcal{ALCI}$

Exactness is a property of *some* interpretations of federated description logics, which ensures seamless propagation of knowledge across importing chains. More precisely, if a concept C in module k is imported by both module i and module j, and module j imports module i, then exactness is equivalent to $r_{kj}(C^k) = r_{ij}(r_{ki}(C^k))$. This has the consequence that, if $\mathcal{I} \models^i C \sqsubseteq D$, then $\mathcal{I} \models^j C \sqsubseteq D$, provided that the interpretation is exact for both concepts C and D. This is a property that may be very desirable in some contexts but not absolutely necessary in others. *Because it imposes rather strong restrictions on the models, we apply it on our interpretations selectively rather than require that it holds universally*, as is done in [2].

Definition 4 (Exactness): Given $(i,j) \in E$, an F-$\mathcal{ALCI}$-interpretation $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$ is said to be (i,j)-**exact** if, for every $C \in \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j$, $r_{ij}(C^i) = C^j$. $\mathcal{I}$ is **exact** if it is (i,j)-exact, for all $(i,j) \in E$.

Note that, in general, the notion of exactness in Definition 4 requires that the condition $r_{ij}(C^i) = C^j$ holds for an infinite collection of concept expressions. For our applications the following weaker concept of exactness, that depends on the contents of a specific knowledge base under consideration, suffices. First let us call a set $\mathcal{E}_i \subseteq \widehat{\mathcal{C}}_i$ of i-concept expressions **closed** if it is closed under concept sub-expressions, i.e., for every $C \in \mathcal{E}_i$, all sub-concepts of C are also in $\mathcal{E}_i$.

Definition 5 (Exactness for T): Let $\mathcal{E} = \{\mathcal{E}_i\}_{i\in V}$, with $\mathcal{E}_i \subseteq \widehat{\mathcal{C}}_i$, $i \in V$, be a V-indexed collection of closed sets of concept expressions and $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$ be an F-$\mathcal{ALCI}$-interpretation. Given $(i,j) \in E$, $\mathcal{I}$ is said to be (i,j)-**exact for** $\mathcal{E}$ if, for every $C \in \mathcal{E}_i \cap \mathcal{E}_j$, $r_{ij}(C^i) = C^j$. $\mathcal{I}$ is **exact for** $\mathcal{E}$ if it is (i,j)-exact for $\mathcal{E}$, for all $(i,j) \in E$.

Let $T = \{T_i\}_{i\in V}$ be an F-$\mathcal{ALCI}$-ontology and $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$ an F-$\mathcal{ALCI}$-interpretation. $\mathcal{I}$ is said to be (i,j)-**exact for** T if it is (i,j)-exact for $\overline{\mathcal{C}} := \{\overline{\mathcal{C}}_i\}_{i\in V}$ and it is said to be **exact for** T if it is (i,j)-exact for T, for all $(i,j) \in E$.

An alternative condition characterizing the exactness of an F-$\mathcal{ALCI}$-interpretation is provided in the following lemma (see [6] for proof).

Lemma 6: An F-$\mathcal{ALCI}$-interpretation $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$ is exact if and only if, for all $k, i, j \in V$, such that $(k,i), (k,j), (i,j) \in E$, $r_{ij}(r_{ki}(C^k)) = r_{kj}(C^k)$, for every $C \in \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j \cap \widehat{\mathcal{C}}_k$. The importing relations are depicted in Figure 2.

$$\begin{tikzcd} & k \arrow[dl, "r_{ki}"'] \arrow[dr, "r_{kj}"] & \\ i \arrow[rr, "r_{ij}"'] & & j \end{tikzcd}$$

Fig. 2. Importing Diagram

A necessary and sufficient condition for the exactness of an F-$\mathcal{ALCI}$-interpretation for a given V-indexed collection $\mathcal{E}$ of closed sets of concept expressions (see [6] for proof).

Lemma 7: Let $\mathcal{E} = \{\mathcal{E}_i\}_{i\in V}$, with $\mathcal{E}_i \subseteq \widehat{\mathcal{C}}_i$, $i \in V$, be a V-indexed collection of closed sets of concept expressions and $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$ an F-$\mathcal{ALCI}$-interpretation. $\mathcal{I}$ is exact for $\mathcal{E}$ if and only if, for all $k, i, j \in V$, such that $(k,i), (k,j), (i,j) \in E$, $r_{ij}(r_{ki}(C^k)) = r_{kj}(C^k)$, for every $C \in \mathcal{E}_i \cap \mathcal{E}_j \cap \mathcal{E}_k$. The importing relations are depicted in Figure 2.

Based on the definition of an exact interpretation, we define exact models of an F-$\mathcal{ALCI}$-ontology.

Definition 8 (Exact Model): Let $T = \{T_i\}_{i\in V}$ be an F-$\mathcal{ALCI}$-ontology. An interpretation $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$ is an **exact model** of T if it is exact for T and $\mathcal{I} \models T$. T is said to be **exactly consistent as witnessed by a module** T_w if there exists an exact model $\mathcal{I}$ of T_w^*, such that $\Delta^w \neq \emptyset$. A concept C is **exactly satisfiable as witnessed by** T_w if there exists an exact model $\mathcal{I}$ of T_w^*, such that $C^w \neq \emptyset$. Finally, a concept subsumption $C \sqsubseteq D$ is **exactly valid as witnessed by** T_w, denoted $C \sqsubseteq_w^e D$ if, for every exact model $\mathcal{I}$ of T_w^*, $C^w \subseteq D^w$. In this case we also write $T_w^* \models_w^e C \sqsubseteq D$.

IV. Decidability of F-$\mathcal{ALCI}$

We establish the decidability of F-$\mathcal{ALCI}$, by providing a **reduction** $\mathfrak{R}$ from an F-$\mathcal{ALCI}$ KB $\Sigma_d = \{T_i\}$ to an $\mathcal{ALCI}$ KB $\Sigma := \mathfrak{R}(\Sigma_d)$: The signature of Σ is the union of the local signatures of the modules together with a global top $\top$, a global bottom $\bot$, local top concepts $\top_i$, for all $i \in V$, and, finally, a collection of new role names $\{R_{ij}\}_{(i,j)\in E}$, i.e.,

$$\begin{aligned} \mathsf{Sig}(\Sigma) \quad = \quad & \textstyle\bigcup_i (\mathcal{C}_i \cup \mathcal{R}_i) \cup \{\top, \bot\} \cup \\ & \{\top_i : 1 \leq i \leq n\} \cup \{R_{ij} : (i,j) \in E\}. \end{aligned}$$

Moreover, various axioms derived from the structure of Σ_d are added to Σ.

- For each $C \in \mathcal{C}_i$, $C \sqsubseteq \top_i$ is added to Σ.

- For each $R \in \mathcal{R}_i$, $\top_i$ is stipulated to be the domain and range of R, i.e., $\top \sqsubseteq \forall R^{-}.\top_i$ and $\top \sqsubseteq \forall R.\top_i$ are added to Σ.
- For each new role name R_{ij}, $\top_i$ is stipulated to be its domain and $\top_j$ to be its range, i.e., $\top \sqsubseteq \forall R_{ij}^{-}.\top_i$ and $\top \sqsubseteq \forall R_{ij}.\top_j$ are added to Σ.
- For each $C \sqsubseteq D \in T_i$, $\#_i(C) \sqsubseteq \#_i(D)$ is added to Σ, where $\#_i$ is a function from $\widehat{\mathcal{C}_i}$ to the set of $\mathcal{ALCI}$-concepts. The precise definition of the mapping $\#_i(C)$, which serves to maintain the compatibility of the concept domains, is obtained by induction on the structure of $C \in \widehat{\mathcal{C}_i}$ e.g., $\#_i(C) = C$, if $C \in \mathcal{C}_i$ etc. (See [6]).

If, in addition to the previous conditions, for each set of importing relations of the form shown in Figure 2 and all $C \in \overline{\mathcal{C}}_i \cap \overline{\mathcal{C}}_j \cap \overline{\mathcal{C}}_k$, $\exists R_{ij}^{-}.(\exists R_{ki}^{-}.\#_k(C)) = \exists R_{kj}^{-}.\#_k(C)$ is added to Σ, then the reduction is said to be an **exact reduction** and is denoted by $\mathfrak{R}_e(\Sigma_d)$.

We have shown that the reduction $\mathfrak{R}$ and its exact counter part $\mathfrak{R}_e$ are *sound* and *complete* [6]:

Theorem 9 (Soundness and Completeness): Suppose that $\Sigma_d = \{T_i\}_{i \in V}$ is an F-$\mathcal{ALCI}$ ontology. Σ_d is consistent as witnessed by a module T_w if and only if $\top_w$ is satisfiable with respect to $\mathfrak{R}(T_w^*)$. Moreover, Σ_d is *exactly* consistent as witnessed by T_w if and only if $\top_w$ is satisfiable with respect to $\mathfrak{R}_e(T_w^*)$.

The language $\mathcal{ALCI}$ is an extension of $\mathcal{ALC}$ and a fragment of $\mathcal{ALCIQ}b$. It is well-known that concept satisfiability, concept subsumption and consistency problems for the language $\mathcal{ALC}$ are PSPACE-complete. The same problems for the language $\mathcal{ALCIQ}b$ are in PSPACE [5]. Hence we have:

Theorem 10: The concept satisfiability, concept subsumption and consistency problems for F-$\mathcal{ALCI}$ are PSPACE-complete.

V. Properties of F-$\mathcal{ALCI}$

A consequence of Theorem 9 is that $\mathfrak{R}$ and its exact counterpart $\mathfrak{R}_e$ are *subsumption-preserving reductions*: That is, a given subsumption is valid as witnessed by a module T_i of an F-$\mathcal{ALCI}$ ontology T if and only if its translation under $\#_i$ is valid with respect to the reduction $\mathfrak{R}(T_i^*)$ (or its *exact* counterpart $\mathfrak{R}_e(T_i^*)$ as the case may be, see [6] for a proof).

Another important consequence of Theorem 9 is the *monotonicity* of federated reasoning *with respect to exact models*. More precisely, we have shown that, given an F-$\mathcal{ALCI}$ ontology $\Sigma_d = \{T_i\}_{i \in V}$ and an exact model $\mathcal{I}_d$ of Σ_d, a subsumption $C \sqsubseteq D$, with $C, D \in \overline{\mathcal{C}}_i \cap \overline{\mathcal{C}}_j$, $(i, j) \in E$, is valid as witnessed by module T_j provided that it is valid as witnessed by module T_i (see [6] for a proof).

It is important to emphasize that this theorem asserts monotonicity of subsumptions in F-$\mathcal{ALCI}$ only for subsumptions between concepts that *actually appear* in modules of the ontology under consideration, and (unlike in the case of P-DLs [1], [2]) *not* for *arbitrary* concept subsumptions (e.g., subsumptions that might be added to the ontology at a later time). This should not be surprising because the exactness conditions imposed in F-$\mathcal{ALCI}$ are considerably milder than the ones imposed on the P-DL semantics.

The *preservation of unsatisfiability* in F-$\mathcal{ALCI}$ follows from *monotonicity*: A concept subsumption $C \sqsubseteq D$, that is unsatisfiable as witnessed by a module T_i, is necessarily unsatisfiable as witnessed by any other module T_j that imports the concepts C, D from T_i.

VI. Summary and Discussion

We have introduced a modular ontology language, the CFDL F-$\mathcal{ALCI}$, which, to the best of our knowledge, represents the first modular ontology language to support contextualized interpretation of *all* logical connectives used within the DL modules. We have established the *decidability* of F-$\mathcal{ALCI}$, which is a prerequisite for automating inference with F-$\mathcal{ALCI}$. In the CFDL F-$\mathcal{ALCI}$, inferences are always drawn *from the point of view* of a *witness* module allowing different modules to infer different consequences, based on the knowledge that they import from other modules. We have characterized the tradeoffs between the restrictions on domain relations (and hence the semantics of the CFDL F-$\mathcal{ALCI}$) and the desirable features of the resulting modular ontologies (e.g., monotonicity of inference and transitive propagation of concept subsumptions across modules linked by importing relations). Work in progress is aimed at exploring more expressive CFDLs as well as developing federated reasoning algorithms for the resulting CFDLs using message passing techniques similar to those introduced for P-DL.

Acknowledgment

This work was supported in part by a grant from the National Science Foundation (IIS 0639230) to Vasant Honavar and Giora Slutzki and in part by an exploratory research grant from the Iowa State University NSF-Industry-University Cooperative Research Center for Information Protection.

References

[1] Jie Bao, Giora Slutzki, and Vasant Honavar. A semantic importing approach to knowledge reuse from multiple ontologies. In *AAAI*, pages 1304–1309, 2007.

[2] Jie Bao, George Voutsadakis, Giora Slutzki, and Vasant Honavar. Package based description logics. In C. Parent, S. Spaccapietra, and H. Stuckenschmidt, editors, *Ontology Modularization*. Springer-Verlag, In Press.

[3] Chiara Ghidini and Fausto Giunchiglia. Local models semantics, or contextual reasoning=locality+compatibility. *Artificial Intelligence*, 127(2):221–259, 2001.

[4] John L. McCarthy. Generality in artificial intelligence. *Commun. ACM*, 30(12):1029–1035, 1987.

[5] Stephan Tobies. Complexity results and practical algorithms for logics in knowledge representation. Ph.d. dissertation, Technischen Hochschule Aachen, Aachen, Germany, 2001.

[6] George Voutsadakis, Jie Bao, Giora Slutzki, and Vasant Honavar. F-$\mathcal{ALCI}$: A fully contextualized, federated logic for the semantic web. Technical report, Iowa State University, Ames, IA, 2008.