PRACTICE FINAL EXAM - MATH 151

DATE: Tuesday, May 5

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Read each problem very carefully before starting to solve it. Each question is worth 20 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

NAME:_____

Section:_____

	Score
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8	
Problem 9	
Problem 10	
TOTAL	

1. (a) Find the vertex, the axis of symmetry, the opening direction and the intercepts and, then, roughly (but clearly) sketch the graph of the function $f(x) = -x^2 - x + 6$ indicating all points collected.

   (b) The price p and the quantity x sold of a certain product obey the demand equation $x = -20p + 500, 0 \leq p \leq 25$.

      i. Express the revenue R as s function of x.

      ii. What is the revenue if 20 units are sold?

      iii. What quantity x maximizes the revenue?

2. (a) Graph the exponential function $f(x) = 3^x$ using the small table of values.

   (b) Solve the exponential equations

   i. $2^x \cdot 16^{-x} = 8^x \cdot 2$.

   ii. $(e^4)^x \cdot e^{x^2} = e^{12}$.

3. (a) Graph the logarithmic function $f(x) = \log_3 x$.

   (b) Solve the following logarithmic equations:

      i. $\log_5 125 = 7x + 31$

      ii. $\log_2 16^x = -8$

   (c) Write as a sum and/or difference of logs: $\log_5 \frac{5x^7 \cdot \sqrt[3]{1-x}}{4(x+9)^2}$.

   (d) Write as a single log: $3\log_2(x+1) - 2\log_2(x+3) - \log_2(x-1)$.

4. (a) Graph the function $f(x) = \begin{cases} e^x - 1, & \text{if } x \leq 0 \\ x^2, & \text{if } x > 0 \end{cases}$. Then find the limit $\lim_{x \to 0} f(x)$ if it exists.

   (b) Find the limit $\lim_{x \to -2} \frac{(x+2)^2}{x^2-4}$. Please, show all your steps.

5. (a) Let $f(x) = \begin{cases} \frac{x^2-2x}{x-2}, & \text{if } x < 2 \\ 2, & \text{if } x = 2 \\ \frac{x-4}{x-1}, & \text{if } x > 2 \end{cases}$ Determine whether $f(x)$ is continuous at $c = 2$. **Justify** your answer.

   (b) Find the derivative of $f(x) = \frac{1}{x^2}$ at $a = 1$ **using the limit definition**.

6. (a) Compute the derivatives of the following functions:
   i. $f(x) = (x^3 - 3)(x^2 + 4)$.
   ii. $f(x) = (x^6 - 2)(4x^2 + 1)$.
   iii. $f(x) = \frac{2x^2-1}{5x+2}$.

   (b) Find an equation for the tangent line to the graph of $y = \frac{x^2}{x-1}$ at the point $(-1, -\frac{1}{2})$.

7. (a) Find the first and the second derivative of each function
   i. $f(x) = xe^x$
   ii. $f(x) = x^2 \ln x$
   (b) Find $\frac{dy}{dx}$ using implicit differentiation:
   i. $x^2y + xy^2 = x + 1$
   ii. $x = \ln(x^2 + y^2)$

8. The weekly revenue R in dollars from selling x calculators is $R(x) = -20x^2 + 1000x$.

   (a) Determine where the graph of R is increasing and where it is decreasing.

   (b) How many calculators have to be solved to maximize revenue?

   (c) What is the maximum revenue?

9. Study fully (find domain, intervals of monotonicity, minima and maxima, intervals of concavity, inflection points, asymptotes) and sketch the graph of the function $f(x) = x^5 + 10x^2 + 2$.

10. (a) Use substitution to evaluate the integrals:
    i. $\int x(x^2-2)^3 dx$
    ii. $\int e^{2x^2+1} x dx$
    (b) Find the area under the graph of $f(x) = \sqrt[3]{x}$ from -8 to 1.
    (c) Find the area under the graph of $f(x) = x^2 - 9$ from 0 to 6.