College Algebra

George Voutsadakis[1]

[1]Mathematics and Computer Science
Lake Superior State University

LSSU Math 111

Subsection 1

Transformations of Graphs and Symmetry

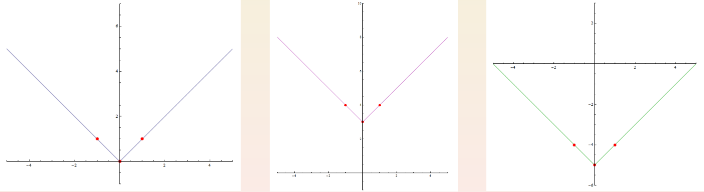
Vertical Shifts

Vertical Shifts of Graphs

Suppose k is a positive real number.

- The graph of $g(x) = f(x) + k$ can be obtained by shifting the graph of $f(x)$ upward by k units.
- The graph of $g(x) = f(x) - k$ can be obtained by shifting the graph of $f(x)$ downward by k units.

Example: Sketch the graph of $f(x) = |x|$.

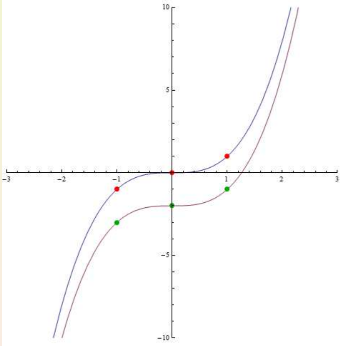


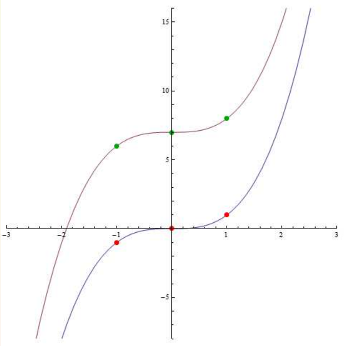
(a) Sketch the graph $f(x) = |x| + 3$.

(b) What is the equation of the graph on the right?
$f(x) = |x| - 5$.

Example of a Vertical Shift

- Sketch the graph of the function $f(x) = x^3$.





(a) Use a vertical shift to graph the function $g(x) = x^3 - 2$.
(b) The graph on the right is that of a shift $h(x)$ of $f(x)$. Find a formula for $h(x)$.
$h(x) = x^3 + 7$.

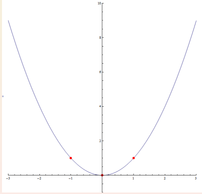
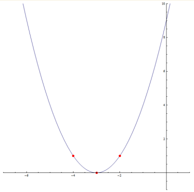
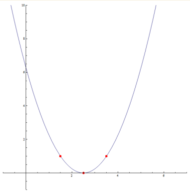
Horizontal Shifts

Horizontal Shifts of Graphs

Suppose h is a positive real number.

- The graph of $g(x) = f(x - h)$ can be obtained by shifting the graph of $f(x)$ to the right h units.
- The graph of $g(x) = f(x + h)$ can be obtained by shifting the graph of $f(x)$ to the left h units.

Example: Sketch the graph of $f(x) = x^2$.

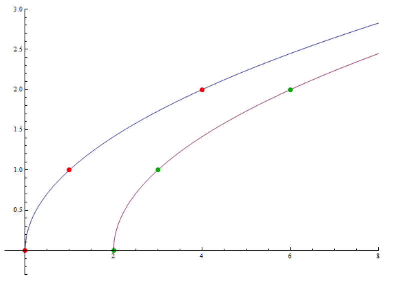




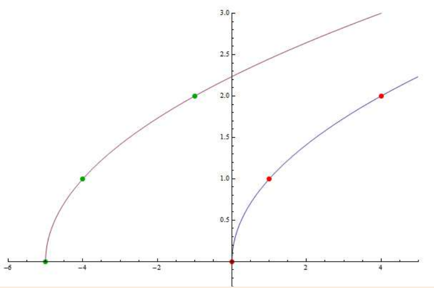
(a) Sketch the graph $f(x) = (x + 3)^2$.

(b) What is the equation of the graph on the right?
$f(x) = (x - \frac{5}{2})^2$.

Example of a Horizontal Shift

- Sketch the graph of the function $f(x) = \sqrt{x}$.



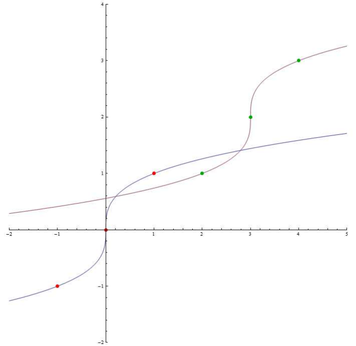


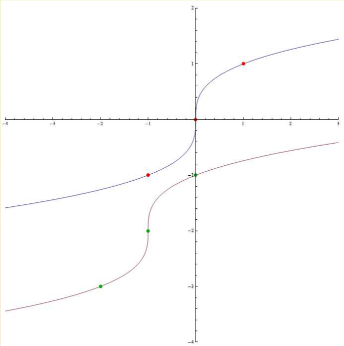
(a) Use a horizontal shift to graph the function $g(x) = \sqrt{x-2}$.

(b) The graph on the right is that of a shift $h(x)$ of $f(x)$. Find a formula for $h(x)$.
$h(x) = \sqrt{x+5}$.

Vertical and Horizontal Shift

- Sketch the graph of the function $f(x) = \sqrt[3]{x}$.





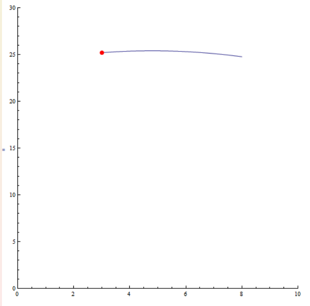
(a) Use vertical and horizontal shifts to graph the function $g(x) = \sqrt[3]{x-3} + 2$.

(b) The graph on the right is that of a shift $h(x)$ of $f(x)$. Find a formula for $h(x)$.
$h(x) = \sqrt[3]{x+1} - 2$.

Marijuana Use

- The number of millions of people age 12 and older in the U.S. who used marijuana during the years 2003 to 2008 is described by the function $M(x) = -0.062(x-4.8)^2 + 25.4$ for $3 \leq x \leq 8$, where x is the number of years after 2000.
  - (a) The graph of this function is a shifted graph of which basic function? It is a shift of $f(x) = x^2$.
  - (b) Find an interpret $M(3)$.

    $M(3) = -0.062(3-4.8)^2 + 25.4 = 25.199$. It signifies that 25,199 people 12 or older used marijuana in 2003.
  - (c) Sketch the graph of $M(x)$ for $3 \leq x \leq 8$.



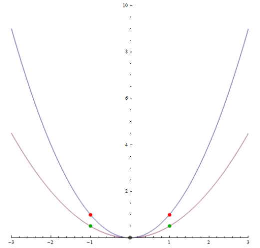
Stretching and Compressing Graphs

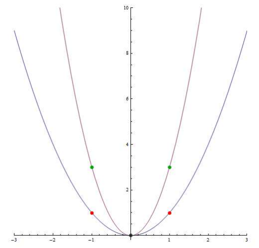
Stretching and Compressing Graphs

The graph of $y = af(x)$ is obtained by vertically stretching the graph of $f(x)$ using a factor of $|a|$, if $|a| > 1$, and vertically compressing the graph of $f(x)$ using a factor of $|a|$, if $0 < |a| < 1$.

Example:

(a) Sketch the graphs of $f(x) = x^2$ and $g(x) = \frac{1}{2}x^2$ on the same system of axes.

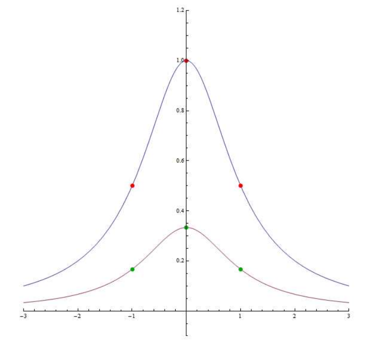


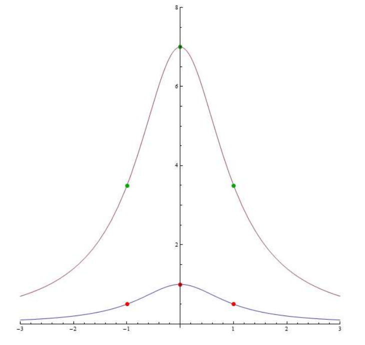


(b) The graph on the right shows $f(x) = x^2$ and which function $h(x)$? $h(x) = 3x^3$.

Stretching and Compressing Graphs

(a) Sketch the graphs of $f(x) = \frac{1}{1+x^2}$ and $g(x) = \frac{1/3}{1+x^2}$ on the same system of axes.





(b) The graph on the right shows $f(x) = \frac{1}{1+x^2}$ and which function $h(x)$?
$h(x) = \frac{7}{1+x^2}$.

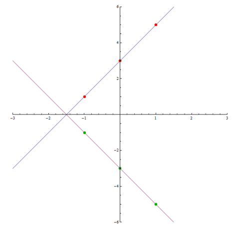
Reflections Across the Coordinate Axes

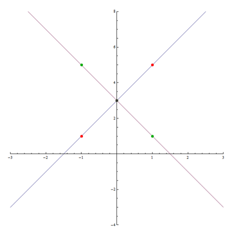
Reflections of Graphs Across the Coordinate Axes

1. The graph of $y = -f(x)$ can be obtained by reflecting the graph of $y = f(x)$ across the x-axis.
2. The graph of $y = f(-x)$ can be obtained by reflecting the graph of $y = f(x)$ across the y-axis.

Example:

(a) Sketch the graphs of $f(x) = 2x + 3$ and $g(x) = -2x - 3$ on the same axes.

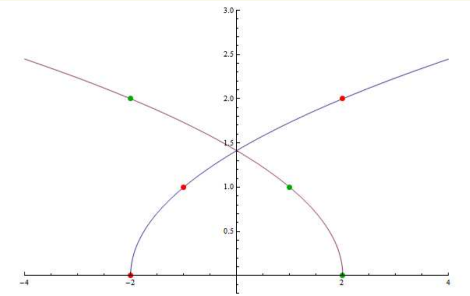


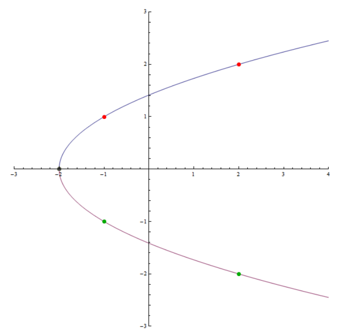


(b) The graph on the right shows $f(x) = 2x + 3$ and which function $h(x)$? $h(x) = f(-x) = 2(-x) + 3 = -2x + 3$.

Reflections Across the Coordinate Axes

(a) Sketch the graphs of $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{-x+2}$ on the same system of axes.





(b) The graph on the right shows $f(x) = \sqrt{x+2}$ and which function $h(x)$?

$h(x) = -f(x) = -\sqrt{x+2}$.

Summary of Transformations

Graph Transformations

For a given function $y = f(x)$:

Vertical Shift: $y = f(x) + k$ The graph is shifted k units up if $k > 0$ and k units down if $k < 0$.

Horizontal Shift: $y = f(x - h)$ The graph is shifted h units right if $h > 0$ and h units left if $h < 0$.

Stretch/Compress: $y = af(x)$ The graph is vertically stretched using a factor of $|a|$ if $|a| > 1$ and compressed using a factor of $|a|$ if $|a| < 1$.

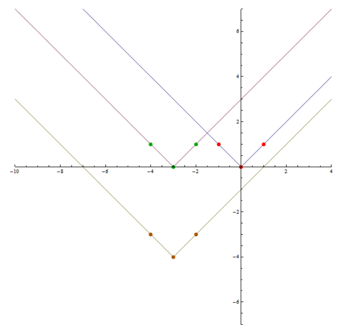
Reflection: $y = -f(x)$ The graph is reflected across the x-axis.

Reflection: $y = f(-x)$ The graph is reflected across the y-axis.

Using Many Transformations I

- Describe the transformations needed to get from $f(x) = |x|$ to $g(x) = |x+3| - 4$ and, then, graph both functions on the same system of axes.
  We follow the following transformations:

$$f(x) = |x| \quad \text{Shift Left 3 Points} \quad y = |x+3|$$
$$\text{Shift Down 4 Points} \quad g(x) = |x+3| - 4.$$

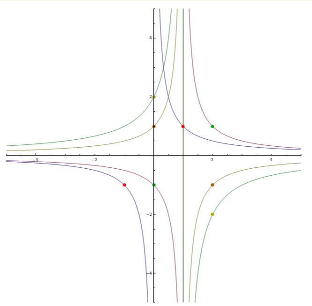


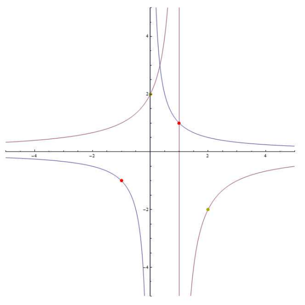
Using Many Transformations II

- Describe the transformations needed to get from $f(x) = \frac{1}{x}$ to $g(x) = \frac{2}{1-x}$ and, then, graph both functions on the same system of axes.

  We follow the following transformations:

$$
\begin{array}{lll}
f(x) = \frac{1}{x} & \text{Shift Right 1 Point} & y = \frac{1}{x-1} \\
 & \text{Reflect w.r.t. the } x\text{-axis} & y = \frac{1}{1-x} \\
 & \text{Vertical Stretch by Factor of 2} & g(x) = \frac{2}{1-x}.
\end{array}
$$

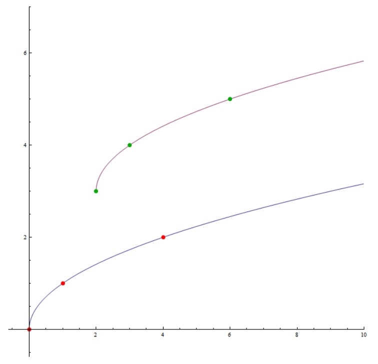




Using Many Transformations III

- The figure below shows the graphs of $f(x) = \sqrt{x}$ and of a function $g(x)$. What is the formula giving $g(x)$?
  We follow the following transformations:

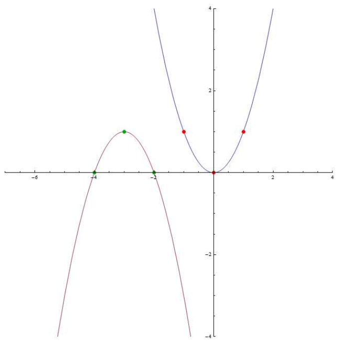
$$f(x) = \sqrt{x} \quad \text{Shift Right 2 Points} \quad y = \sqrt{x-2}$$
$$\text{Shift Up 3 Points} \quad g(x) = \sqrt{x-2} + 3.$$



Using Many Transformations IV

- The figure below shows the graphs of $f(x) = x^2$ and of a function $g(x)$. What is the formula giving $g(x)$?
  We follow the following transformations:

$f(x) = x^2$	Shift Left 3 Points	$y = (x+3)^2$
	Reflect w.r.t. x-axis	$y = -(x+3)^2$
	Shift Up 1 Point	$g(x) = -(x+3)^2 + 1.$

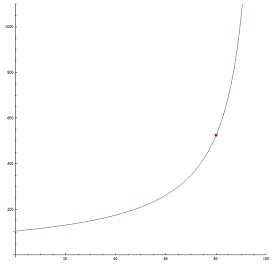


Pollution

- The daily cost C in dollars of removing pollution from the smokestack of a coal-fired electric power plant is related to the percent of pollution p being removed according to the equation $C = \frac{10{,}500}{100-p}$.
  (a) Describe the transformations needed to obtain this function from the function $C = \frac{1}{p}$.

$C = \frac{1}{p}$	Shift Right 100 Points	$C = \frac{1}{p-100}$
	Reflect w.r.t. x-axis	$C = \frac{1}{100-p}$
	Stretch by a Factor of 10500	$C = \frac{10{,}500}{100-p}$.

  (b) Graph the function for $0 \leq p < 100$.



  (c) What is the daily cost of removing 80% of the pollution?

$$C(80) = \frac{10{,}500}{100-80} = \$525.$$

Cost-Benefit

- Suppose for a certain city the cost C of obtaining drinking water that contains p% impurities by volume is given by $C = \frac{120{,}000}{p} - 1200$.
  (a) What is the cost of drinking water that is 100% impure?
  $C(100) = \frac{120{,}000}{100} - 1200 = \$0$.
  (b) What is the cost of drinking water that is 50% impure?
  $C(50) = \frac{120{,}000}{50} - 1200 = \$1,200$.
  (c) What transformations of the graph of the reciprocal function give the graph of this function?

$C = \frac{1}{p}$ Stretch by a Factor of 120000 $C = \frac{120000}{p}$

Shift Down by 1200 Points $C = \frac{120{,}000}{p} - 1200$.

Even Functions

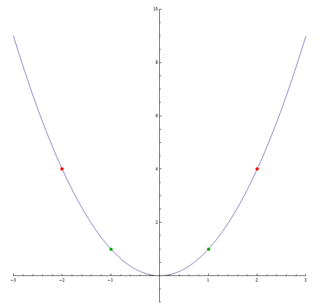
Symmetry with respect to the y-axis

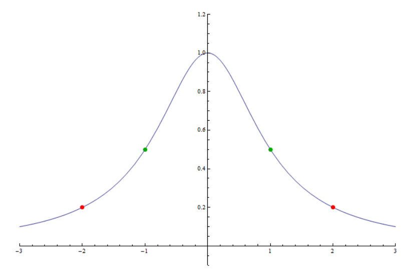
The graph of $y = f(x)$ is **symmetric with respect to the y-axis** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. In other words,

$$f(-x) = f(x),$$

for all in the domain of x. Such a function is called an **even function**.

Examples: $f(x) = x^2$ and $g(x) = \frac{1}{1+x^2}$.





Odd Functions

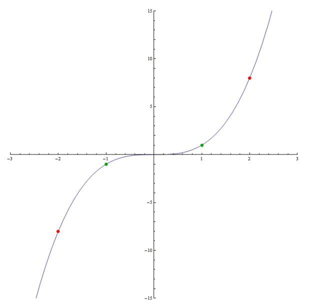
Symmetry with respect to the Origin

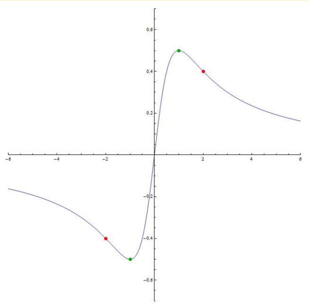
The graph of $y = f(x)$ is **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. In other words,

$$f(-x) = -f(x),$$

for all in the domain of x. Such a function is called an **odd function**.

Examples: $f(x) = x^3$ and $g(x) = \frac{x}{1+x^2}$.



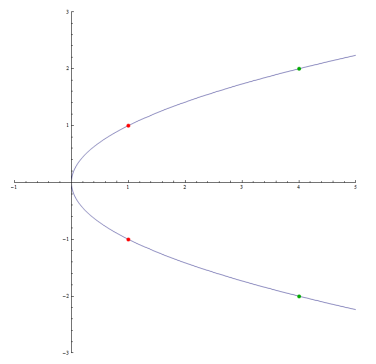


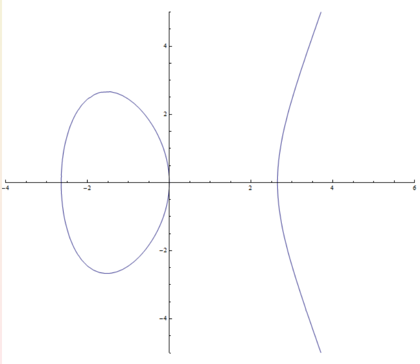
Symmetry with respect to the x-axis

Symmetry with respect to the x-axis

The graph of an equation is **symmetric with respect to the x-axis** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

Examples: $y^2 = x$ and $y^2 = x^3 - 7x$.





Checking for Symmetry I

- Determine algebraically whether the graph of $y = \frac{2x^2}{x^2+1}$ is symmetric with respect to the x-axis, y-axis or origin.
  - For x-axis symmetry, replace y by $-y$ and test for equivalence:
  $$-y = \frac{2x^2}{x^2+1} \Leftrightarrow y = -\frac{2x^2}{x^2+1} \not\Leftrightarrow y = \frac{2x^2}{x^2+1}.$$
  Thus, $y = \frac{2x^2}{x^2+1}$ is not symmetric with respect to the x-axis.
  - For y-axis symmetry, replace x by $-x$ and test for equivalence:
  $$y = \frac{2(-x)^2}{(-x)^2+1} \Leftrightarrow y = \frac{2x^2}{x^2+1}.$$
  Thus, $y = \frac{2x^2}{x^2+1}$ is symmetric with respect to the y-axis.
  - For origin symmetry, replace x by $-x$ and y by $-y$ and test for equivalence:
  $$-y = \frac{2(-x)^2}{(-x)^2+1} \Leftrightarrow y = -\frac{2x^2}{x^2+1} \not\Leftrightarrow y = \frac{2x^2}{x^2+1}.$$
  Thus, $y = \frac{2x^2}{x^2+1}$ is not symmetric with respect to the origin.

Checking for Symmetry II

- Determine algebraically whether the graph of $y = x^3 - 3x$ is symmetric with respect to the x-axis, y-axis or origin.
  - For x-axis symmetry, replace y by $-y$ and test for equivalence:
    $$-y = x^3 - 3x \Leftrightarrow y = -x^3 + 3x \not\Leftrightarrow y = x^3 - 3x.$$
    Thus, $y = x^3 - 3x$ is not symmetric with respect to the x-axis.
  - For y-axis symmetry, replace x by $-x$ and test for equivalence:
    $$y = (-x)^3 - 3(-x) \Leftrightarrow y = -x^3 + 3x \not\Leftrightarrow y = x^3 - 3x.$$
    Thus, $y = x^3 - 3x$ is not symmetric with respect to the y-axis.
  - For origin symmetry, replace x by $-x$ and y by $-y$ and test for equivalence:
    $$-y = (-x)^3 - 3(-x) \Leftrightarrow -y = -x^3 + 3x \Leftrightarrow y = x^3 - 3x.$$
    Thus, $y = x^3 - 3x$ is symmetric with respect to the origin.

Checking for Symmetry III

- Determine algebraically whether the graph of $x^2 + y^2 = 16$ is symmetric with respect to the x-axis, y-axis or origin.
  - For x-axis symmetry, replace y by $-y$ and test for equivalence:
    $$x^2 + (-y)^2 = 16 \Leftrightarrow x^2 + y^2 = 16.$$
    Thus, $x^2 + y^2 = 16$ is symmetric with respect to the x-axis.
  - For y-axis symmetry, replace x by $-x$ and test for equivalence:
    $$(-x)^2 + y^2 = 16 \Leftrightarrow x^2 + y^2 = 16.$$
    Thus, $x^2 + y^2 = 16$ is symmetric with respect to the y-axis.
  - For origin symmetry, replace x by $-x$ and y by $-y$ and test for equivalence:
    $$(-x)^2 + (-y)^2 = 16 \Leftrightarrow x^2 + y^2 = 16.$$
    Thus, $x^2 + y^2 = 16$ is symmetric with respect to the origin as well.

Subsection 2

Combining Functions and Composition

Operations with Functions

Operations with Functions

Operation	Formula	$f=\sqrt{x}, g=x^3$
Sum	$(f+g)(x)=f(x)+g(x)$	$(f+g)(x)=\sqrt{x}+x^3$
Difference	$(f-g)(x)=f(x)-g(x)$	$(f-g)(x)=\sqrt{x}-x^3$
Product	$(fg)(x)=f(x)g(x)$	$(fg)(x)=x^3\sqrt{x}$
Quotient	$(\frac{f}{g})(x)=\frac{f(x)}{g(x)}$	$(\frac{f}{g})(x)=\frac{\sqrt{x}}{x^3}$

Example: Let $f(x)=2x^2-x$ and $g(x)=2x+1$. Find the following functions:

(a) $(f+g)(x)=2x^2-x+2x+1=2x^2+x+1$;
(b) $(f-g)(x)=2x^2-x-(2x+1)=2x^2-x-2x-1=2x^2-3x-1$;
(c) $(fg)(x)=(2x^2-x)(2x+1)=4x^3+2x^2-2x^2-x=4x^3-x$;
(d) $(\frac{f}{g})(x)=\frac{2x^2-x}{2x+1}$;

Further Examples

- Let $f(x) = x^3$ and $g(x) = \sqrt{x+3}$. Find the following functions:
  (a) $(f+g)(x) = f(x) + g(x) = x^3 + \sqrt{x+3}$;
  (b) $(f-g)(x) = f(x) - g(x) = x^3 - \sqrt{x+3}$;
  (c) $(fg)(x) = f(x)g(x) = x^3\sqrt{x+3}$;
  (d) $(\frac{f}{g})(x) = \frac{f(x)}{g(x)} = \frac{x^3}{\sqrt{x+3}}$;
- If $f(x) = 4 - x^2$ and $g(x) = x^3 + x$, evaluate the following:
  (a) $(f+g)(1) = f(1) + g(1) = (4 - 1^2) + (1^3 + 1) = 3 + 2 = 5$;
  (b) $(f-g)(-2) = f(-2) - g(-2) = 4 - (-2)^2 - [(-2)^3 + (-2)] = 0 - (-10) = 10$;
  (c) $(fg)(-3) = f(-3)g(-3) = [4 - (-3)^2][(-3)^3 + (-3)] = (-5)(-30) = 150$;
  (d) $(\frac{g}{f})(2) = \frac{g(2)}{f(2)} = \frac{2^3+2}{4-2^2} =$ Undefined!!

Profit

- Suppose that the total weekly cost for the production and sale of TV sets is $C(x) = 189x + 5460$ and that the total revenue is given by $R(x) = 988x$, where x is the number of TV sets and $C(x)$ and $R(x)$ are in dollars.
  (a) Write the equation of the function that models the weekly profit from the production and sale of x TV sets.

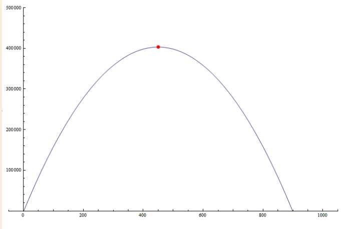
$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 988x - (189x + 5460) \\ &= 988x - 189x - 5460 \\ &= 799x - 5460. \end{aligned}$$

  (b) What is the profit on the production and sale of 80 TV sets in a given week?

$$P(80) = 799 \cdot 80 - 5460 = \$58,460.$$

Revenue and Cost

- The total monthly revenue function for camcorders is given by $R = 6600x$ dollars and the total monthly cost function is $C = 2000 + 4800x + 2x^2$ dollars, where x is the number of camcorders that are produced and sold.
  (a) Find the profit function.
  $P(x) = R(x) - C(x) = 6600x - (2000 + 4800x + 2x^2) = -2x^2 + 1800x - 2000.$
  (b) Find the number of camcorders that gives maximum profit.



The maximum occurs when $x = 450$ camcorders are produced and sold.

  (c) Find the maximum possible profit.
  The max profit is $P(450) = -2 \cdot 450^2 + 1800 \cdot 450 - 2000 = \$403,000.$

Average Cost: A Greenhouse Application

Average Cost Function

A company's **average cost** per unit, when x units are produced, is the quotient of the function $C(x)$ (total production cost) over the number x of the units produced:

$$\overline{C}(x) = \frac{C(x)}{x}.$$

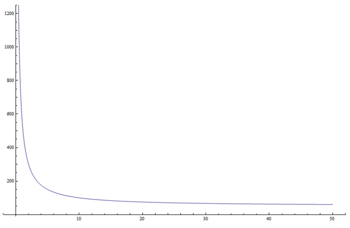
Example: Amy's Greenhouse produces roses and their total cost for the production of x hundred roses is $C(x) = 50x + 500$.

(a) Form the average cost function.

$\overline{C}(x) = \frac{C(x)}{x} = \frac{50x+500}{x}$.

(b) For which input values is $\overline{C}(x)$ defined?

$\overline{C}(x)$ is only defined for $x > 0$.



(c) Graph $\overline{C}$ for 0 to 5000 roses (50 units) and describe the average cost. As the number of roses increases, the average cost per rose decreases.

Average Cost for Printers

- The weekly total cost function for producing a dot matrix printer is

$$C(x) = 3000 + 72x,$$

where x is the number of printers produced per week.

(a) Form the weekly average cost function for this product.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{3000 + 72x}{x}.$$

(b) Find the average cost for the production of 100 printers.

$$\overline{C}(100) = \frac{3000 + 72 \cdot 100}{100} = \frac{10200}{100} = \$102.$$

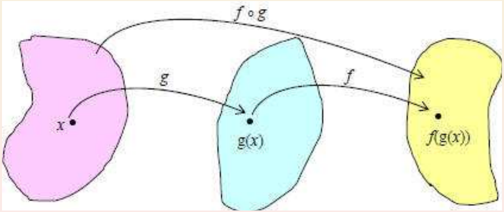
Composition of Functions

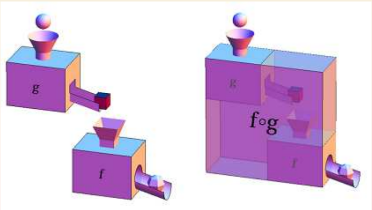
Composite Functions

The **composite function** f **of** g is denoted by $f \circ g$ and defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the subset of the domain of g for which $f \circ g$ is defined.





Examples of Composition of Functions

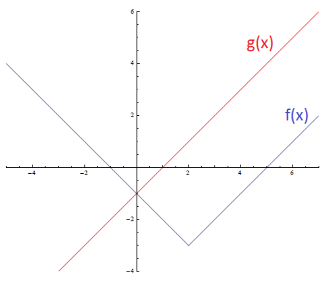
- Let $f(x) = 2x - 5$, $g(x) = 6 - x^2$ and $h(x) = \frac{1}{x}$. Find the following composite functions and provide the domain of each.
  (a) $(h \circ f)(x) = h(f(x)) = h(2x-5) = \frac{1}{2x-5}$.
  The domain is $\mathrm{Dom}(h \circ f) = \mathbb{R} - \{\frac{5}{2}\}$.
  (b) $(f \circ g)(x) = f(g(x)) = f(6-x^2) = 2(6-x^2)-5 = 12-2x^2-5 = 7-2x^2$.
  The domain is $\mathrm{Dom}(h \circ f) = \mathbb{R}$.
  (c) $(g \circ f)(x) = g(2x-5) = 6-(2x-5)^2 = 6-(4x^2-20x+25) = -4x^2+20x-19$.
  The domain is $\mathrm{Dom}(g \circ f) = \mathbb{R}$.
- Let $f(x) = x^3$ and $g(x) = \frac{2}{x}$. Find the following composite functions and provide the domain of each.
  (a) $(f \circ g)(x) = f(g(x)) = f(\frac{2}{x}) = (\frac{2}{x})^3 = \frac{8}{x^3}$.
  The domain is $\mathrm{Dom}(f \circ g) = \mathbb{R} - \{0\}$.
  (b) $(g \circ f)(x) = g(f(x)) = g(x^3) = \frac{2}{x^3}$.
  The domain is $\mathrm{Dom}(g \circ f) = \mathbb{R} - \{0\}$.

More Examples of Composition

- Let $f(x)=\sqrt{3-x}$ and $g(x)=x-5$. Find the following composite functions and provide the domain of each.
  - (a) $(f\circ g)(x)=f(g(x))=f(x-5)=\sqrt{3-(x-5)}=\sqrt{8-x}$.
    The domain is $\text{Dom}(f\circ g)=(-\infty,8]$.
  - (b) $(g\circ f)(x)=g(f(x))=g(\sqrt{3-x})=\sqrt{3-x}-5$.
    The domain is $\text{Dom}(g\circ f)=(-\infty,3]$.
- Let $f(x)=\sqrt[3]{x+1}$ and $g(x)=x^3+1$. Find the following composite functions and provide the domain of each.
  - (a) $(f\circ g)(x)=f(g(x))=f(x^3+1)=\sqrt[3]{(x^3+1)+1}=\sqrt[3]{x^3+2}$.
    The domain is $\text{Dom}(f\circ g)=\mathbb{R}$.
  - (b) $(g\circ f)(x)=g(f(x))=g(\sqrt[3]{x+1})=(\sqrt[3]{x+1})^3+1=x+1+1=x+2$.
    The domain is $\text{Dom}(g\circ f)=\mathbb{R}$.

Evaluating Expressions

- Let $f(x) = (x-1)^2$ and $g(x) = 3x - 1$. Evaluate the following expressions:
  (a) $(f \circ g)(2) = f(g(2)) = f(3 \cdot 2 - 1) = f(5) = (5-1)^2 = 16.$
  (b) $(g \circ f)(-2) = g(f(-2)) = g((-2-1)^2) = g(9) = 3 \cdot 9 - 1 = 26.$
- Use the following graphs of f and g to evaluate the following expressions:



  (a) $(g-f)(-2) = g(-2) - f(-2) = -3 - 1 = -4.$
  (b) $(f \circ g)(3) = f(g(3)) = f(2) = -3.$
  (c) $(\frac{f}{g})(0) = \frac{f(0)}{g(0)} = \frac{-1}{-1} = 1.$
  (d) $(f \circ g)(-2) = f(g(-2)) = f(-3) = 2.$

Harvesting

- A farmer's cash crop is tomatoes and the tomato harvest begins in May. The number of bushels of tomatoes harvested on the x-th day of May is given by the equation $B(x) = 6(x+1)^{3/2}$. The market price in dollars of 1 bushel of tomatoes on the x-th day of May is given by the formula $P(x) = 8.5 - 0.12x$.
  (a) How many bushels did the farmer harvest on May 8?
  $$B(8) = 6(8+1)^{3/2} = 6 \cdot 9^{3/2} = 6 \cdot 27 = 162 \text{ bushels.}$$
  (b) What was the market price of tomatoes on May 8?
  $$P(8) = 8.5 - 0.12 \cdot 8 = 8.5 - 0.96 = \$7.54/\text{bushel.}$$
  (c) How much was the farmer's tomato harvest worth on May 8?
  $$B(8) \cdot P(8) = 7.54 \cdot 162 = \$1,221.48.$$
  (d) Write a model for the worth W of the tomato harvest on the x-th day of May.
  $$W(x) = B(x) \cdot P(x) = (6(x+1)^{3/2})(8.5 - 0.12x) \text{ dollars.}$$

Profit

- A manufacturer of computers has monthly fixed costs of \$87,500 and variable costs of \$87 per computer and sells the computers for \$295 per unit.
  (a) Write the function that models the profit P from the production and sale of x computers.

$$\begin{aligned} R(x) &= 295x \\ C(x) &= 87x + 87500 \\ P(x) &= R(x) - C(x) = 295x - (87x + 87500) \\ &= 208x - 87500. \end{aligned}$$

  (b) What is the profit if 700 computers are produced and sold in 1 month?
  $P(700) = 208 \cdot 700 - 87500 = \$58,100.$
  (c) What is the y-intercept of the graph of the profit function? What does it mean?
  $P(0) = -87500$. It signifies the initial investment before production starts.

Shoe Sizes

- A man's shoe that is size x in Britain is size $d(x)$ in the U.S., where $d(x) = x + 0.5$. A man's shoe that is size x in the U.S. is size $t(x)$ in Continental size, where $t(x) = x + 34.5$. Find a function that will convert British shoe size to Continental shoe size.

$$\begin{aligned} t(d) &= t(d(x)) \\ &= t(x + 0.5) \\ &= (x + 0.5) + 34.5 \\ &= x + 35. \end{aligned}$$

Exchange Rates

- On March 11, 2011, each euro was worth 1.3773 U.S. dollars and each Mexican peso was worth 0.06047 euro. Find the value of 100 Mexican pesos in U.S. dollars.

$$\begin{aligned} d(\text{Mex\$}100) &= d(e(\text{Mex\$}100)) \\ &= d(€0.06047/\text{Mex\$} \cdot \text{Mex\$}100) \\ &= d(€6.047) \\ &= \$1.3773/€ \cdot €6.047 \\ &= \$8.33. \end{aligned}$$

Subsection 3

One-to-One and Inverse Functions

Inverse Functions

Example: Consider the two functions expressing the temperature in $^\circ$F in terms of the temperature in $^\circ$C and vice-versa:

$$F(x) = \frac{9}{5}x + 32 \quad \text{and} \quad C(x) = \frac{5x - 160}{9}.$$

It holds that $C(F(x)) = x$ and $F(C(x)) = x$.

Inverse Functions

Functions f and g for which $f(g(x)) = x$, for all x in the domain of g, and $g(f(x)) = x$, for all x in the domain of f, are called **inverse functions**. In this case, we denote g by f^{-1}, read "f inverse".

Example: Check that F and C are inverse functions, using the definition.

$$C(F(x)) = C(\tfrac{9}{5}x + 32) = \tfrac{5(\frac{9}{5}x+32)-160}{9} = \tfrac{9x+160-160}{9} = \tfrac{9x}{9} = x;$$

$$F(C(x)) = F(\tfrac{5x-160}{9}) = \tfrac{9}{5}(\tfrac{5x-160}{9}) + 32 = x - 32 + 32 = x.$$

Examples of Inverse Functions

- Suppose $f(x) = 4x - 1$ and $g(x) = \frac{x+1}{4}$.
  (a) What are $f(g(x))$ and $g(f(x))$?

$$f(g(x)) = f(\tfrac{x+1}{4}) = 4(\tfrac{x+1}{4}) - 1 = x + 1 - 1 = x;$$
$$g(f(x)) = g(f(x)) = g(4x-1) = \tfrac{(4x-1)+1}{4} = \tfrac{4x}{4} = x.$$

  (b) Are $f(x)$ and $g(x)$ inverse functions?
  Yes, because $f(g(x)) = x$ and $g(f(x)) = x$.
- Suppose $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$.
  (a) What are $f(g(x))$ and $g(f(x))$?

$$f(g(x)) = f(\sqrt[3]{x-1}) = (\sqrt[3]{x-1})^3 + 1 = x - 1 + 1 = x;$$
$$g(f(x)) = g(x^3+1) = \sqrt[3]{(x^3+1)-1} = \sqrt[3]{x^3} = x.$$

  (b) Are $f(x)$ and $g(x)$ inverse functions?
  Yes, because $f(g(x)) = x$ and $g(f(x)) = x$.

Value of Inverse Functions

- Suppose that the following tables refer to the function $f(x) = 2x^3 + 5$ and its inverse function f^{-1}. Complete the missing values:

x	$f(x)$
-2	-11
-1	3
0	5
1	6
2	21

x	$f^{-1}(x)$
-11	-2
3	-1
5	0
6	1
21	2

One-to-One Functions

One-to-One Function

A function f is **one-to-one** if each output corresponds to exactly one input in the domain of the function. This means that there is a one-to-one correspondence between the elements of the domain and the elements of the range.

- The following condition states that f is one-to-one if it holds for all a, b in the domain of f:

$$a \neq b \quad \text{implies} \quad f(a) \neq f(b).$$

Example: Determine whether $f(x) = 3x^4$ and $g = x^3 - 1$ are one-to-one functions.

  - We have $-1 \neq 1$, but $f(-1) = 3 = f(1)$. Thus, to two different values in the domain, there corresponds a single value in the range. This shows that f is not one-to-one.
  - If $a \neq b$, then $a^3 \neq b^3$ and, therefore, $g(a) = a^3 - 1 \neq b^3 - 1 = g(b)$. This shows that g is one-to-one.

More Examples of One-to-One Functions

- Determine whether the following functions are one-to-one.
  (a) $\{(1,5),(2,6),(3,7),(4,5)\}$
  The function is not one to one, since to $1 \neq 4$ there corresponds the same image 5.
  (b) $f(x) = \frac{1}{x}$
  If $a \neq b$, then $f(a) = \frac{1}{a} \neq \frac{1}{b} = f(b)$. Therefore, f is one-to-one.
  (c) $f(x) = -2x^4$ Note that $f(-1) = -2 = f(1)$. Hence f is not one-to-one.
  (d) $f(x) = \sqrt{x+3}$.
  Suppose that $a \neq b$. Then, $a+3 \neq b+3$, which implies that $f(a) = \sqrt{a+3} \neq \sqrt{b+3} = f(b)$. Therefore, f is one-to-one.

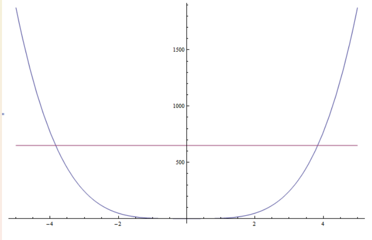
Horizontal Line Test

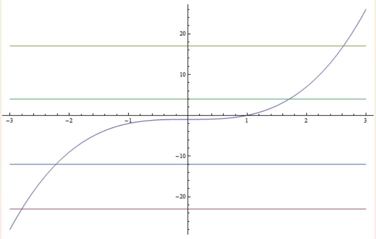
Horizontal Line Test

A function is one-to-one if no horizontal line can intersect the graph of the function in more than one point.

Example: Determine using the horizontal line test whether each of the following functions is one-to-one.

(a) $f(x) = 3x^4$





So, the function f is not one-to-one.

(b) $g(x) = x^3 - 1$.

Thus, the function g is one-to-one.

Domains and Ranges of Inverse Functions

Domains and Ranges of Inverse Functions

The functions f and g are inverse functions if, whenever the pair (a, b) satisfies $y = f(x)$, the pair (b, a) satisfies $y = g(x)$. Therefore, the domain of the function f is the range of its inverse g and the domain of g is the range of f.

Example:

(a) Determine if $f(x) = x^3 - 1$ has an inverse function.
Since f is one-to-one, it has an inverse function.

(b) Verify that $g(x) = \sqrt[3]{x+1}$ is the inverse function of $f(x) = x^3 - 1$.
We have

$$f(g(x)) = f(\sqrt[3]{x+1}) = (\sqrt[3]{x+1})^3 - 1 = x + 1 - 1 = x;$$
$$g(f(x)) = g(x^3 - 1) = \sqrt[3]{(x^3 - 1) + 1} = \sqrt[3]{x^3} = x.$$

(c) Find the domain and range of each function.
The domain of f is $\mathbb{R}$. Therefore, the range of g is also $\mathbb{R}$. The domain of g is $\mathbb{R}$, whence the range of f is $\mathbb{R}$.

Finding the Inverse of a Function

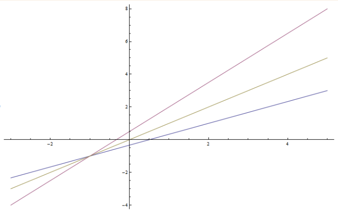
Finding the Inverse of a Function

To find the inverse of a function f that is defined by $y = f(x)$:

1. Rewrite the equation replacing $f(x)$ by y.
2. Interchange x and y in the equation.
3. Solve the equation for y. If the equation is not uniquely solvable for y, then the original function does not have an inverse.
4. Replace y by $f^{-1}(x)$.

Example: Find the inverse function of $f(x) = \frac{2x-1}{3}$ and graph $y = f(x)$ and its inverse.

$$f(x) = \frac{2x-1}{3} \Rightarrow y = \frac{2x-1}{3}$$
$$\rightsquigarrow x = \frac{2y-1}{3} \Rightarrow 3x = 2y - 1$$
$$\Rightarrow 2y = 3x + 1 \Rightarrow y = \frac{3x+1}{2}$$
$$\Rightarrow f^{-1}(x) = \frac{3x+1}{2}.$$



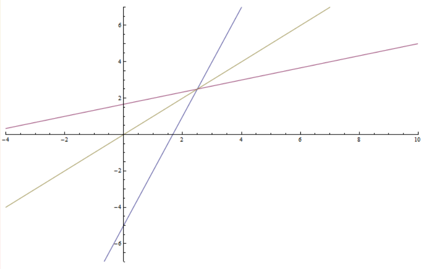
Graphs of Inverse Functions

Graphs of Inverse Functions

The graph of a function and its inverse are symmetric with respect to the line $y = x$.

Example: Find the inverse function of $f(x) = 3x - 5$ and graph both on the same system of axes.

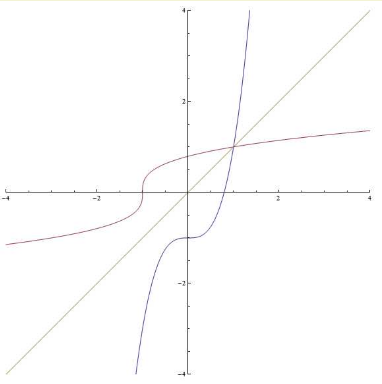
$$f(x) = 3x - 5 \Rightarrow y = 3x - 5$$
$$\rightsquigarrow x = 3y - 5 \Rightarrow 3y = x + 5$$
$$\Rightarrow y = \frac{x+5}{3} \Rightarrow f^{-1}(x) = \frac{x+5}{3}.$$



Example of Graphs of Inverse Functions

Example: Find the inverse function of $f(x) = 2x^3 - 1$ and graph both on the same system of axes.

$$f(x) = 2x^3 - 1 \Rightarrow y = 2x^3 - 1$$
$$\rightsquigarrow x = 2y^3 - 1 \Rightarrow 2y^3 = x + 1$$
$$\Rightarrow y^3 = \frac{x+1}{2} \Rightarrow y = \sqrt[3]{\frac{x+1}{2}}$$
$$\Rightarrow f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}.$$



Loan Repayment

- A business property is purchased with a promise to pay off a \$60,000 loan plus the \$16,500 interest on this loan by making 60 monthly payments of \$1275. The amount of money remaining to be paid on the loan plus interest is given by the function $f(x) = 76,500 - 1275x$, where x is the number of monthly payments remaining.
  (a) Find the inverse of the function.

$$\begin{aligned} & f(x) = 76,500 - 1275x \quad \Rightarrow \quad y = 76,500 - 1275x \\ & \rightsquigarrow \quad x = 76,500 - 1275y \quad \Rightarrow \quad 1275y = 76500 - x \\ & \Rightarrow \quad y = \frac{76500-x}{1275} \quad \Rightarrow \quad f^{-1}(x) = \frac{76500-x}{1275}. \end{aligned}$$

  (b) Use the inverse to determine how many monthly payments remain if \$35,700 remains to be paid.

$$f^{-1}(35700) = \frac{76500-35700}{1275} = \frac{40800}{1275} = 32.$$

Cigarettes

- For the years 1997-2009, the percent of high school seniors who have tried cigarettes is given by $f(x) = 82.074 - 2.087x$, where x is the number of years after 1990.
  (a) Find the inverse of this function.

$$\begin{array}{l} f(x) = 82.074 - 2.087x \quad \Rightarrow \quad y = 82.074 - 2.087x \\ \rightsquigarrow \quad x = 82.074 - 2.087y \quad \Rightarrow \quad 2.087y = 82.074 - x \\ \Rightarrow \quad y = \frac{82.074 - x}{2.087} \quad \Rightarrow \quad f^{-1}(x) = \frac{82.074 - x}{2.087}. \end{array}$$

  (b) Use it to find the year in which the percent fell below 41%.

$$f^{-1}(41) = \frac{82.074 - 41}{2.087} = 2010.$$

Body-Heat Loss

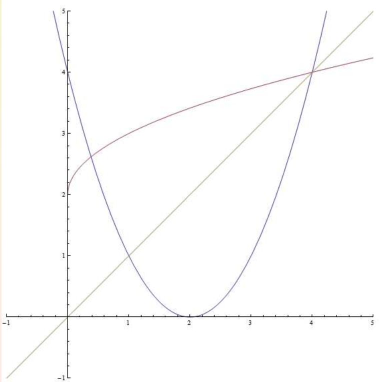
- The model for body-heat loss depends on the coefficient of convection $K = f(x)$, which depends on wind speed x according to the equation $f(x) = 4\sqrt{4x+1}$.
  (a) What are the domain and the range of this function without regard to the context of the application?
  The domain is found by setting $4x+1 \geq 0$. This yields $4x \geq -1$, whence $x \geq -\frac{1}{4}$. The range is $y \geq 0$.
  (b) Find the inverse of this function.

$$\begin{array}{lllll} f(x) = 4\sqrt{4x+1} & & \Rightarrow & & y = 4\sqrt{4x+1} \\ \rightsquigarrow & x = 4\sqrt{4y+1} & \Rightarrow & & \sqrt{4y+1} = \frac{x}{4} \\ \Rightarrow & 4y+1 = \frac{1}{16}x^2 & \Rightarrow & & 4y = \frac{1}{16}x^2 - 1 \\ \Rightarrow & y = \frac{1}{64}x^2 - \frac{1}{4} & \Rightarrow & & f^{-1}(x) = \frac{x^2-16}{64}. \end{array}$$

  (c) What are the domain and range of the inverse function?
  The domain if $\mathbb{R}$. The range is $y \geq -\frac{1}{4}$.
  (d) In the context of the application, what are the domain and range of the inverse function?
  We must have $x \geq 4$ and $f^{-1}(x) \geq 0$.

Restricting the Domain

- For which values of x are $f(x) = (x-2)^2$ and $g(x) = \sqrt{x} + 2$ inverse functions?



Note that $f(x)$ is not one-to-one on $\mathbb{R}$. It is only one-to-one on the domain $[2, +\infty)$. On that domain its range is $[0, +\infty)$.

Thus, for $x \geq 2$, f and g are inverse functions and the domain of g is $[0, +\infty)$, whereas its range is $[2, +\infty)$.

Surface Area

- The surface area of a cube is $f(x) = 6x^2$ cm^2, where x is the length of the edge of the cube in centimeters.
  (a) For which values of x does this model make sense?
  The model makes sense for $x \geq 0$.
  (b) Is the model one-to-one for these values of x?
  The model is one-to-one for $x \geq 0$.
  (c) What is the inverse function on this interval?

$$\begin{array}{lll} f(x) = 6x^2 & \Rightarrow & y = 6x^2 \\ \rightsquigarrow \quad x = 6y^2 & \Rightarrow & y^2 = \frac{x}{6} \\ \Rightarrow \quad y = \sqrt{\frac{x}{6}} & \Rightarrow & f^{-1}(x) = \sqrt{\frac{x}{6}}. \end{array}$$

  (d) How could the inverse function be used?
  It computes the length in centimeters of the edge of a cube with a given surface area in square centimeters.

Supply

- The supply function for a product is $p(x) = \frac{1}{4}x^2 + 20$, where x is the number of thousands of units a manufacturer will supply if the price is $p(x)$ dollars.
  (a) Is this function a one-to-one function?
  It is not a one-to-one function.
  (b) What is the domain of this function in the context of the application?
  We must have $x \geq 0$.
  (c) Is the function one-to-one for the domain in Part (b)?
  For $x \geq 0$, the function is one-to-one.
  (d) Find the inverse of this function and use it to find how many units the manufacturer is willing to supply if the price is \$101.

$$\begin{array}{ll} p(x) = \frac{1}{4}x^2 + 20 & \Rightarrow \quad y = \frac{1}{4}x^2 + 20 \\ \rightsquigarrow \quad x = \frac{1}{4}y^2 + 20 & \Rightarrow \quad x - 20 = \frac{1}{4}y^2 \\ \Rightarrow \quad 4(x-20) = y^2 & \Rightarrow \quad y = 2\sqrt{x-20} \\ \Rightarrow \quad f^{-1}(x) = 2\sqrt{x-10}. & \end{array}$$

$f^{-1}(101) = 2\sqrt{101-20} = 2\sqrt{81} = 18$ thousand units.

Subsection 4

Additional Equations and Inequalities

Solving Radical Equations

Solving Radical Equations

1. Isolate a single radical on one side of the equation.
2. Raise both sides to a power equal to the index of the radical.
3. If a radical remains repeat Steps 1 and 2.
4. Solve the resulting equation.
5. All solutions must be checked in the original equation, and only those that satisfy it are admissible.

Example: Solve the radical equation $\sqrt{x+5}+1=x$

$$\begin{aligned}
&\sqrt{x+5}=x-1 \quad \Rightarrow \quad (\sqrt{x+5})^2=(x-1)^2\\
&\Rightarrow \quad x+5=x^2-2x+1 \quad \Rightarrow \quad x^2-3x-4=0\\
&\Rightarrow \quad (x-4)(x+1)=0 \quad \Rightarrow \quad x=-1 \text{ or } x=4.
\end{aligned}$$

Only $x=4$ is admissible.

Another Radical Equation

- Solve the radical equation

$$\sqrt{3x-2}+2=x.$$

$$\begin{aligned}
&\sqrt{3x-2}+2=x\\
\Rightarrow\quad &\sqrt{3x-2}=x-2\\
\Rightarrow\quad &(\sqrt{3x-2})^2=(x-2)^2\\
\Rightarrow\quad &3x-2=x^2-4x+4\\
\Rightarrow\quad &x^2-7x+6=0\\
\Rightarrow\quad &(x-1)(x-6)=0\\
\Rightarrow\quad &x=1 \text{ or } x=6.
\end{aligned}$$

$x=6$ is the only admissible solution.

A Radical Equation Involving Two Radicals

- Solve the radical equation

$$\sqrt{4x-8}-1=\sqrt{2x-5}.$$

$$\begin{aligned} &\sqrt{4x-8}-1=\sqrt{2x-5} \\ \Rightarrow &\quad (\sqrt{4x-8}-1)^2=(\sqrt{2x-5})^2 \\ \Rightarrow &\quad 4x-8-2\sqrt{4x-8}+1=2x-5 \\ \Rightarrow &\quad 2\sqrt{4x-8}=2x-2 \\ \Rightarrow &\quad \sqrt{4x-8}=x-1 \\ \Rightarrow &\quad (\sqrt{4x-8})^2=(x-1)^2 \\ \Rightarrow &\quad 4x-8=x^2-2x+1 \\ \Rightarrow &\quad x^2-6x+9=0 \\ \Rightarrow &\quad (x-3)^2=0 \\ \Rightarrow &\quad x=3. \end{aligned}$$

$x=3$ is an admissible solution.

An Equation With Rational Powers

- Solve the equation
$$(x-3)^{2/3}-4=0.$$

$$\begin{aligned}
&(x-3)^{2/3}-4=0\\
\Rightarrow\quad & \sqrt[3]{(x-3)^2}=4\\
\Rightarrow\quad & (x-3)^2=4^3\\
\Rightarrow\quad & x-3=\pm\sqrt{64}\\
\Rightarrow\quad & x-3=-8 \text{ or } x-3=8\\
\Rightarrow\quad & x=-5 \text{ or } x=11.
\end{aligned}$$

Both $x=-5$ and $x=11$ are admissible.

Another Equation With Rational Powers

- Solve the equation
$$(x-5)^{3/2}=64.$$

$$\begin{aligned}
&(x-5)^{3/2}=64\\
&\Rightarrow \quad (\sqrt{x-5})^3=64\\
&\Rightarrow \quad \sqrt{x-5}=\sqrt[3]{64}=4\\
&\Rightarrow \quad x-5=4^2=16\\
&\Rightarrow \quad x=21.
\end{aligned}$$

$x=21$ is an admissible solution.

Solving a Quadratic Inequality Algebraically

Solving a Quadratic Inequality Algebraically

1. Write an inequality with 0 on one side and $f(x)$ on the other.
2. Solve $f(x) = 0$.
3. Create the sign table for $f(x)$ using the solutions from Step 2.
4. Identify the intervals that satisfy the inequality in Step 1.

Example: Solve the inequality $x^2 - 3x > 3 - 5x$.
$x^2 - 3x > 3 - 5x$ implies $x^2 + 2x - 3 > 0$. This is equivalent to $(x+3)(x-1) > 0$. The roots of $(x+3)(x-1) = 0$ are $x = -3$, $x = 1$. Using these, we create the sign table for the expression $x^2 + 2x - 3$.

	$x < -3$	$-3 < x < 1$	$1 < x$
$x^2 + 2x - 3$	$+$	$-$	$+$

Since we want $x^2 + 2x - 3 > 0$, we pick the intervals with the "+" signs. Thus the solution set is $x < -3$ or $x > 1$, or, in interval notation, $x \in (-\infty, -3) \cup (1, \infty)$.

Another Quadratic Inequality

- Solve the inequality

$$x^2 + 17x \leq 8x - 14.$$

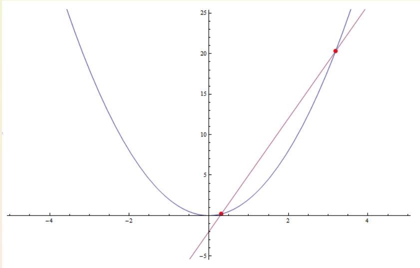
$x^2 + 17x \leq 8x - 14$ implies $x^2 + 9x + 14 \leq 0$. This is equivalent to $(x+7)(x+2) \leq 0$. Thus, the roots of $(x+7)(x+2) = 0$ are $x = -7$, $x = -2$. Using these, we create the sign table for the expression $x^2 + 9x + 14$.

	$x < -7$	$-7 < x < -2$	$-2 < x$
$x^2 + 9x + 14$	$+$	$-$	$+$

Since we want $x^2 + 9x + 14 \leq 0$, we pick the intervals with the "$-$" signs plus the zeros. Thus the solution set is $-7 \leq x \leq -2$, or, in interval notation, $x \in [-7, -2]$.

Solving a Quadratic Inequality Graphically

- Solve the following inequality graphically:
$$2x^2 \geq 7x - 2.$$
We graph the functions $y = 2x^2$ and $y = 7x - 2$ and find the points of intersection.

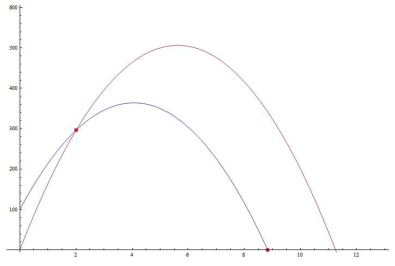


We get $x = \frac{7 \pm \sqrt{33}}{4}$. Since we want $2x^2 \geq 7x - 2$, we find the interval where the parabola lies above the straight line. This happens outside the two points. Thus, the solution set is $x \leq \frac{7-\sqrt{33}}{4}$ or $x \geq \frac{7+\sqrt{33}}{4}$, or, in interval notation, $x \in (-\infty, \frac{7-\sqrt{33}}{4}] \cup [\frac{7+\sqrt{33}}{4}, \infty)$.

Projectiles

- Two projectiles are fired into the air over a lake with the height of the first projectile given by $y = 100 + 130t - 16t^2$ and the height of the second projectile given by $y = -16t^2 + 180t$, where y is in feet and t in seconds. Over what time interval, before the lower one hits the lake, is the second projectile above the first?

  We graph $y = 100 + 130t - 16t^2$ and $y = -16t^2 + 180t$ and find:



- The point of intersection:

  $t = 2$;

- The point when the lower projectile falls into the lake.

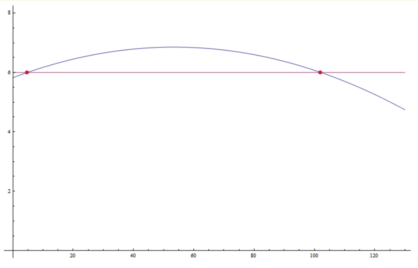
  $t = 8.83$.

Thus, the second projectile is above the first for $2 < t < 8.83$ seconds.

World Population

- The low long-range world population numbers and projections for the years 1995-2150 are given by $y = -0.00036x^2 + 0.0385x + 5.823$, where x is the number of years after 1990 and y is in billions. During what years does this model estimate the population to be above 6 billion?

  We graph $y = -0.00036x^2 + 0.0385x + 5.823$ and $y = 6$ and find the points if intersection.

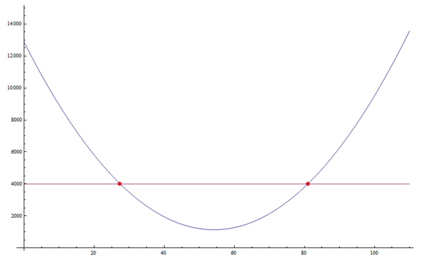


We get $x = 5$ and $x = 102$. Thus, the population will be above 6 billion from 1995 to 2092.

Gross Domestic Product

- The U.S. gross domestic product (in billions of constant dollars) can be modeled by the equation $y = 3.99x^2 - 432.50x + 12,862.21$, where x is the number of years after 1900. During what years prior to 2010 was the gross domestic product less than $4 trillion?

  We graph $y = 3.99x^2 - 432.50x + 12,862.21$ and $y = 4,000$ and find the points if intersection.



We get $x = 28$ and $x = 80$. Thus, the gross domestic product is less than $4 trillion from 1928 to 1980.

Power Inequalities

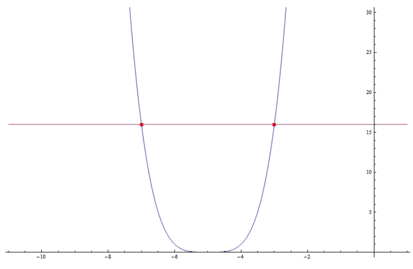
Power Inequalities

To solve a power inequality, we first solve the related equation and, then, use graphical methods to find the values of the variable that satisfy the inequality.

Example: Solve $(x+5)^4 > 16$.

We have

$$\begin{aligned} & (x+5)^4 > 16 \\ \Rightarrow \quad & x+5 = \pm\sqrt[4]{16} \\ \Rightarrow \quad & x+5 = -2 \text{ or } x+5 = 2 \\ \Rightarrow \quad & x = -7 \text{ or } x = -3. \end{aligned}$$



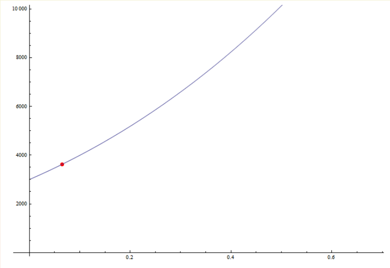
By graphing, we see that $(x+5)^4 > 16$ when $x < -7$ or $x > -3$.

Investment

- The future value of \$3,000 invested for 3 years at rate r, compounded annually, is given by $S = 3000(1+r)^3$. What interest rate will give a future value of at least \$3630?

  We have

$$3000(1+r)^3 = 3630$$
$$\Rightarrow \quad (1+r)^3 = 1.21$$
$$\Rightarrow \quad 1+r = \sqrt[3]{1.21} \approx 1.0656$$
$$\Rightarrow \quad r = 0.0656.$$



  By graphing, we see that $3000(1+r)^3 \geq 3630$ when $r \geq 0.0656$, i.e., the interest rate must be at least 6.56%.

Absolute Value Inequalities

Absolute Value Inequalities

For $a \geq 0$:

- $|u| < a$ means $-a < u < a$.
- $|u| \leq a$ means $-a \leq u \leq a$.
- $|u| > a$ means $u < -a$ or $u > a$.
- $|u| \geq a$ means $u \leq -a$ or $u \geq a$.

Example: Solve the inequality $|2x-3| \leq 5$.

$$|2x-3| \leq 5 \quad \Rightarrow \quad -5 \leq 2x-3 \leq 5$$
$$\Rightarrow \quad -2 \leq 2x \leq 8 \quad \Rightarrow \quad -1 \leq x \leq 4.$$

Example: Solve the inequality $|3x+4|-5>0$.

$$|3x+4|-5>0 \quad \Rightarrow \quad |3x+4|>5$$
$$\Rightarrow \quad 3x+4<-5 \text{ or } 3x+4>5$$
$$\Rightarrow \quad 3x<-9 \text{ or } 3x>1$$
$$\Rightarrow \quad x<-3 \text{ or } x>\frac{1}{3}.$$

Voltage

- Required voltage for an electric oven is 220 volts, but it will function normally if the voltage varies from 220 by 10 volts.
  (a) Write an absolute value inequality that gives the voltage x for which the oven will work normally.
  $$|x - 220| \leq 10.$$
  (b) Solve this inequality for x.
  $$\begin{aligned} & |x - 220| \leq 10 \\ \Rightarrow \quad & -10 \leq x - 220 \leq 10 \\ \Rightarrow \quad & 210 \leq x \leq 230. \end{aligned}$$
  Thus, normal operation is assured if the voltage is maintained between 210 and 230 volts.