EXAM 2: SOLUTIONS - MATH 110
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Problem 1 *Find the point of intersection of the line that goes through the points* $(1,3)$ *and* $(4,9)$ *and of the line that is perpendicular to it and goes through* $(2,\frac{15}{2})$.

Solution:

The line that goes through the points $(1,3)$ and $(4,9)$ has slope

$$m=\frac{9-3}{4-1}=\frac{6}{3}=2.$$

Hence its equation is given by the point-slope form as

$$y-3=2(x-1), \quad \text{i.e.,} \quad y=2x+1.$$

On the other hand, the line that is perpendicular to it has slope $-\frac{1}{2}$ and goes through the point $(2,\frac{15}{2})$. Hence its equation is given by the point-slope form as

$$y-\frac{15}{2}=-\frac{1}{2}(x-2), \quad \text{i.e.,} \quad y-\frac{15}{2}=-\frac{1}{2}x+1.$$

Therefore its equation is $y=-\frac{1}{2}x+\frac{17}{2}$.

Now, the point of intersection of these two lines is given by the solution of the 2×2 system

$$\left\{\begin{array}{rcl} y & = & 2x+1 \\ y & = & -\frac{1}{2}x+\frac{17}{2} \end{array}\right\}$$

We have

$$2x+1=-\frac{1}{2}x+\frac{17}{2}, \quad \text{implies} \quad \frac{5}{2}x=\frac{15}{2},$$

whence $x=3$. Thus $y=2\cdot 3+1=7$. The point of intersection is therefore the point $(3,7)$. ■

Problem 2 *Two competing food stores* Al's Food *and* Bill's Co-op *sell the same brand of wine for \$3 and \$4.50 per bottle, respectively. Both stores gross the same revenue from the sales of this wine but* Bill's Co-op *sells 20 bottles less than* Al's Food*. Can you compute the revenue of the two groceries from the sales of this wine?*

Solution:

Let x be the number of bottles sold by Al's Food and y the number of bottles sold by Bill's co-op. Since both stores gross the same revenue from the sales of the wine, we must have $3x=4.5y$. On the other hand, Bill's co-op sells 20 bottles less than Al's Food, whence $y=x-20$. Thus, we have

$$3x=4.5(x-20), \quad \text{i.e.,} \quad 3x=4.5x-90,$$

whence $1.5x=90$ and, therefore, $x=60$. This gives $y=40$. The revenue of each grocery thus from the sales of this brand of wine is $R=3\cdot 60=\$180$. ■