EXAM 3: SOLUTIONS - MATH 110 INSTRUCTOR: George Voutsadakis

Problem 1 (a) Use truth tables to show that $P \to Q \equiv \neg P \lor Q$.

- (b) Use (a) and De Morgan's laws to rewrite the negation of the statement "If Maria lives in Athens, then she lives in Greece".
- (c) Use truth tables to show that the following argument form is valid

$$\begin{array}{c} P \to Q \\ Q \to R \\ \hline P \to R \end{array}$$

(d) Use (c) to fill-in the statement that is missing below, to make the following argument valid

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Solution:

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(a)

P	Q	$ \neg P$	$P \to Q$	$\neg P \lor Q$
F	F	T	Т	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

Since the last two columns agree on all rows we have $P \to Q \equiv \neg P \lor Q$.

(b) Let P = "Maria lives in Athens" and Q = "Maria lives in Greece". Then, the given sentence is the sentence $P \to Q$ This is equivalent by (a) to $\neg P \lor Q$. Its negation is then equivalent to $\neg(\neg P \lor Q)$ which, by de Morgan's Law is equivalent to $\neg \neg P \land \neg Q$, i.e., to $P \land \neg Q$. Therefore the given sentence is equivalent to

"Maria lives in Athens and Maria does not live in Greece".

(c)

P	Q	R	$P \rightarrow Q$	$Q \to R$	$P \to R$
F	F	F	Т	T	T
F	F	T	T	T	T
F	T	F	Т	F	T
F	T	T	Т	T	T
T	F	F	F	T	F
T	F	T	F	T	F
T	T	F	T	F	T
T	T	T	T	T	T

In all critical rows, the conclusion is true, whence the given argument form is valid.

(d)

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Problem 2 Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:

- (a) Socko: Lefty killed Sharky.
- (b) Fats: Muscles did not kill Sharky.
- (c) Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.
- (d) Muscles: Lefty did not kill Sharky.

Who killed Sharky?

Solution:

Let L = "Lefty killed Sharky" and M = "Muscles killed Sharky". Then we have, for the four given statements to detective Sharp:

- (a) Socko: L.
- (b) Fats: $\neg M$.
- (c) Lefty: $\neg M$.
- (d) Muscles: $\neg L$.

Build the truth table for those sentences

L	M	L	$\neg M$	$\neg M$	$\neg L$
F	F	F	Т	Т	T
F	T	F	F	F	T
T	F	T	T	T	F
T	T	T	F	F	F

Only in the second and fourth rows are three of the statements true and one false. And in the fourth row both Lefty and Muscles are the killers, which is not possible. Hence, only the second row is valid, which means that Muscles is the killer.