EXAM 4: SOLUTIONS - MATH 110
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Problem 1 *(a) Is it true that for all sets A, B, C, if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$? If yes,* **give a formal proof***. If no,* **give a counterexample***.*

(b) In this problem, for two sets X, Y, we write $X \not\subseteq Y$ for "it is not the case that $X \subseteq Y$". Is it true that for all sets A, B, C if $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$? If yes, **give a formal proof***. If no,* **give a counterexample***.*

Solution:

(a) Draw a Venn diagram to get an idea about the truth or falsity of the given statement. The given statement is true! We prove it as follows:

We need to show that, if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$. So suppose that $A \subseteq C$ and $B \subseteq C$ and let $x \in A \cup B$. We need to show that $x \in C$. By the definition of union $x \in A$ or $x \in B$. So consider the following two cases

(i) If $x \in A$, then, since $A \subseteq C$, we must also have $x \in C$.

(ii) If $x \in B$, then, since $B \subseteq C$, we must also have $x \in C$.

Therefore in either of the two possible cases we have $x \in C$. We have thus shown that $x \in A \cup B$ implies $x \in C$, i.e., that $A \cup B \subseteq C$.

(b) Draw a Venn diagram to get an idea about the truth or falsity of the given statement. The given statement is not true! Here is a counterexample:

Let $U = \{0, 1\}$. Take $A = \{0\}, B = \{1\}$ and $C = \{0\}$. Then we have $A = \{0\} \not\subseteq \{1\} = B$ and $B = \{1\} \not\subseteq \{0\} = C$ but $A = \{0\} \subseteq \{0\} = C$.

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Problem 2 *(a) (i) In how many ways can the letters of the word "EIGHT" be arranged in a row?*

(ii) In how many ways can the letters of "EIGHT" be arranged in a row if G and H must remain together (in order) as a unit?

(b) (i) How many 16-bit strings contain exactly nine 1's?

(ii) How many 16-bit strings contain at least fourteen 1's?

(ii) How many 16-bit strings contain at least one 1?

Solution:

(a) (i) $P(5, 5) = 5!$

(ii) $P(4, 4) = 4!$

(b) (i) $\binom{16}{9} = \frac{16!}{9!7!}$.

(ii) $\binom{16}{14} + \binom{16}{15} + \binom{16}{16}$.

(ii) $2^{16} - 1$

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