

## HOMEWORK 4 - MATH 110

DUE DATE: Friday, October 4

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each problem is worth 3 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Using results discussed in Section 2.3 estimate the number of prime numbers that are less than  $10^{10}$ .

(**Important:** Your instructor made a mistake when presenting this result in class. So, please, look at your books and correct your lecture notes before solving.)

2. Does there exist a number  $n$  so that both  $n$  and  $n + 1$  are prime numbers? If so, find such an  $n$ . If not, show why not.
3. Which of the following is the correct UPC for Canada Dry tonic water? Show why the other numbers are not valid UPCs.

0 1 6 9 0 0 0 0 3 0 3 4,

0 2 4 0 0 1 1 0 6 9 1 3,

0 1 0 0 1 0 2 0 1 1 0 5

4. In our discussion the two public numbers 7 and 143 were given. How would you encode the word “2”? The secret decoding number is 103. Without performing the calculation, how would you decode the encrypted message you just made if you are the receiver?
5. Recall how exponents work, for example,  $7^{15} = 7^{(12+3)} = 7^{12} \times 7^3$ . Now, using exponent antics, compute  $5^{668} \pmod{7}$ .
6. Show that  $\sqrt{5}$  is irrational.