HOMEWORK 5: SOLUTIONS - MATH 110 INSTRUCTOR: George Voutsadakis

Problem 1 Solve the equation 2x - 5 = 7x - 6.

Solution:

We have 2x - 5 = 7x - 6 implies, by adding -2x + 6 to both sides, 2x - 5 - 2x + 6 = 7x - 6 - 2x + 6 which yields 1 = 5x. By multiplying both sides by $\frac{1}{5}$, we get $x = \frac{1}{5}$.

Problem 2 Two speedboats start racing on a straight line at speeds of 30 mph and 45 mph, respectively. If the first needs 2 hours more to reach the destination point, how far away is the destination point from the starting point?

Solution:

The time t_1 that the first (slower) speedboat needs to cover the total distance x is given by $t_1 = \frac{x}{30}$. Similarly, the time t_2 that the second speedboat needs to cover the distance x is $t_2 = \frac{x}{45}$. We know that

$$t_1 = t_2 + 2$$
, i.e., $\frac{x}{30} = \frac{x}{45} + 2$.

Now multiply both sides by 90, to get 3x = 2x + 180, which yields x = 180. Thus the destination point is 180 miles away from the starting point.

Problem 3 You are attending a party where the organizers are offering two kinds of booze. One has alcohol content 4 % and the second has alcohol content 12 %. A superstitious friend of yours that likes his liquor containing exactly 7 % asks you to mix him a 16 oz glass from the two drinks containing exactly 7%. How many oz of each liquor do you need?

Solution:

Suppose that we need to mix x oz of the first drink (the 4%) and 16 - x oz of the second drink (the 12%). Then the amount of alcohol in the first drink will be $x \cdot 0.04$ and the amount of alcohol in the second drink will be $(16 - x) \cdot 0.12$. The total amount of alcohol in the final mix will be $16 \cdot 0.07$. Thus, since the last drink contains the alcohol from the first drink plus the alcohol from the second drink, we must have

$$x \cdot 0.04 + (16 - x) \cdot 0.12 = 16 \cdot 0.07.$$

We proceed to solve this for x. We have 0.04x + 1.92 - 0.12x = 1.12, i.e., -0.08x = -0.8 which gives x = 10 oz. Thus we need 10 oz of the first and 6 oz of the second drink to create the final mix.

Problem 4 Solve the equation 6y = 8x + 12.

Solution:

We have $y = \frac{8}{6}x + \frac{12}{6}$, i.e., $y = \frac{4}{3}x + 2$. Hence the solutions are

$$(x, \frac{4}{3}x+2), \quad x \quad \text{any real number.}$$

Problem 5 Find the equation of the line that goes through the points (-1,3) and (5,-3).

Solution:

First, we determine the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 3}{5 - (-1)} = \frac{-6}{6} = -1.$$

Now we use the point-slope form y - b = m(x - a) to obtain y - 3 = -1(x - (-1)), i.e., y - 3 = -x - 1, whence y = -x + 2.

Problem 6 Find the equation of the line that is perpendicular to the line going through (5,1) and (3,-2) and goes through the point (2,1).

Solution:

Since the given line goes through the points (5, 1) and (3, -2), its slope is given by

$$\frac{-2-1}{3-5} = \frac{-3}{-2} = \frac{3}{2}.$$

The unknown line is perpendicular to the given one. So its slope must be the negative inverse of $\frac{3}{2}$. Thus $m = -\frac{2}{3}$. Since it goes through the point (2, 1), the point-slope form may be used and gives $y - 1 = -\frac{2}{3}(x - 2)$, i.e., $y - 1 = -\frac{2}{3}x + \frac{4}{3}$, whence $y = -\frac{2}{3}x + \frac{7}{3}$.