HOMEWORK 8: SOLUTIONS - MATH 110
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Problem 1 *Let $N =$ "Olga will go out for tennis" and $K =$ "Olga will go out for track". Write in symbolic logical form the English statement "Either Olga will go out for tennis or she will go out for track but not both".*

Solution:

The English "either or but not both" is captured by the exclusive or connective $\oplus$. In Example 68, it was seen that $P \oplus Q \equiv (P \vee Q) \wedge \neg(P \wedge Q)$. The given statement, which is, thus, $N \oplus K$ may equivalently be written as $(N \vee K) \wedge \neg(N \wedge K)$. ■

Problem 2 *Construct the truth table for the statement form $\neg P \wedge (Q \vee \neg R)$.*

Solution:

We have

P	Q	R	$\neg P$	$\neg R$	$Q \vee \neg R$	$\neg P \wedge (Q \vee \neg R)$
F	F	F	T	T	T	T
F	F	T	T	F	F	F
F	T	F	T	T	T	T
F	T	T	T	F	T	T
T	F	F	F	T	T	F
T	F	T	F	F	F	F
T	T	F	F	T	T	F
T	T	T	F	F	T	F

■

Problem 3 *Determine whether $P \vee (P \wedge Q) \equiv P$.*

Solution:

We have

P	Q	$P \wedge Q$	$P \vee (P \wedge Q)$	P
F	F	F	F	F
F	T	F	F	F
T	F	F	T	T
T	T	T	T	T

Thus, since the columns for $P \vee (P \wedge Q)$ and P agree, we have $P \vee (P \wedge Q) \equiv P$. ■

Problem 4 *Determine whether $(P \wedge Q) \vee R \equiv P \wedge (Q \vee R)$.*

Solution:

We have

P	Q	R	$P \wedge Q$	$Q \vee R$	$(P \wedge Q) \vee R$	$P \wedge (Q \vee R)$
F	F	F	F	F	F	F
F	F	T	F	T	T	F
F	T	F	F	T	F	F
F	T	T	F	T	T	F
T	F	F	F	F	F	F
T	F	T	F	T	T	T
T	T	F	T	T	T	T
T	T	T	T	T	T	T

The columns of $(P \wedge Q) \vee R$ and $P \wedge (Q \vee R)$ differ in the second and fourth rows. Therefore, $(P \wedge Q) \vee R \not\equiv P \wedge (Q \vee R)$. ■

Problem 5 *Use De Morgan's Laws to write the negation for the statement "Sam swims on Thursdays and Kate plays tennis on Saturdays".*

Solution:
Let

$$P = \text{“Sam swims on Thursdays”} \quad \text{and} \quad Q = \text{“Kate plays tennis on Saturdays”}.$$

Then the given statement is $P \wedge Q$. Hence, its negation is the statement $\neg(P \wedge Q)$. This is by De Morgan's law logically equivalent to the statement $\neg P \vee \neg Q$. Thus, the negation of the previous statement is the statement “Sam does not swim on Thursdays or Kate does not play tennis on Saturdays”. ■

Problem 6 *Find out whether the following statement form is a tautology or a contradiction*

$$(\neg P \vee Q) \vee (P \wedge \neg Q).$$

Solution:
We have

P	Q	$\neg P$	$\neg Q$	$\neg P \vee Q$	$P \wedge \neg Q$	$(\neg P \vee Q) \vee (P \wedge \neg Q)$
F	F	T	T	T	F	T
F	T	T	F	T	F	T
T	F	F	T	F	T	T
T	T	F	F	T	F	T

Since, no matter which are the truth values of P and Q, the statement $(\neg P \vee Q) \vee (P \wedge \neg Q)$ always assumes the value T, it is a tautological statement. ■