FINAL EXAM - MATH 111
Wednesday, December 18, 2002
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 4 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the point of intersection of the line that goes through the points $(1, 3)$ and $(4, 9)$ and of the line that is perpendicular to it and goes through $(2, \frac{15}{2})$.

2. Find the domain of the function $f(x) = \sqrt{\frac{-x+2}{x+5}}$.

3. Find the vertex, the opening direction, the x- and y-intercepts and sketch the graph of $f(x) = -3x^2 - 6x$.

4. Find the equation of the parabola that has vertex $V = (4, 1)$ and goes through the point $(0, 2)$.

5. Solve the equations

   (a) $5^{x^2-7} = 125^{2x}$.

   (b) $\log_2 (x + 1) - \log_2 (x - 5) = 2$.

6. Solve the following system **by the Gauss-Jordan method**

$$\left\{ \begin{array}{rrrrrrr} x & + & y & + & z & = & 0 \\ -2x & - & y & + & z & = & 5 \\ x & + & 2y & - & 2z & = & -1 \end{array} \right\}.$$

7. An urn contains 3 red, 5 black, 2 white and 7 green marbles. Two marbles are drawn at random without replacement. Find the probability of

   (a) the first marble being white and the second being green,

   (b) one marble being black and one red.

8. In a U.S. state, 20% of the population lives in inner cities, 35% in suburbs and 45% in rural areas. 20% of those living in inner cities receive poor medical care and the corresponding probabilities for those living in the suburbs and in rural areas are 5% and 10%, respectively. A person in the population selected at random receives satisfactory medical care. What is the probability that he came from the inner cities?

9. A committee of the United Nations consists of 6 Chinese, 5 Indian, 3 American, 2 Canadian and 4 European members.

   (a) A subcommittee of 5 is to be formed on Asian affairs. In how many ways can such a subcommittee be formed if it is to consist of 2 Chinese, 1 Indian and 2 non-Asian members?

   (b) A subcommittee of 8 is to be formed consisting of a Chairman, a Vice-Chairman, a Secretary and 5 members. In how many ways can such a subcommittee be formed?

10. A pair of fair dice are rolled 11 times. Find the probabilities that

   (a) sum 7 appears at least once.
   (b) sum 5 appears at most twice.