HOMEWORK 3: SOLUTIONS - MATH 111
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Problem 1 *The equation of the line perpendicular to* $y = 5x - 2$ *and passing through* $(5, 2)$ *is*

$$(a) \quad y = -\frac{1}{5}x + 1 \quad (b) \quad y = 5x + 2 \quad (c) \quad y = -\frac{1}{5}x + 3 \quad (d) \quad y = \frac{1}{5}x + 1$$

Solution:

Since the unknown line is perpendicular to $y = 5x - 2$, its slope m will be such that $5m = -1$, i.e., $m = -\frac{1}{5}$. Then, since it passes through $(5, 2)$, the point-slope form gives $y - 2 = -\frac{1}{5}(x - 5)$, whence $y - 2 = -\frac{1}{5}x + 1$ or $y = -\frac{1}{5}x + 3$. Hence (c) is the correct answer. ■

Problem 2 *The equation of the line with* x*-intercept* 3 *and* y*-intercept* -9 *is*

$$(a) \quad y = 2x - 9 \quad (b) \quad y = -9x + 3 \quad (c) \quad y = -\frac{1}{3}x - 1 \quad (d) \quad y = 3x - 9$$

Solution:

The given points are $(3, 0)$ and $(0, -9)$. The slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-9 - 0}{0 - 3} = 3.$$

Since the y-intercept is $b = -9$, the equation is given by the slope-intercept form as $y = 3x - 9$. Hence (d) is the correct answer. ■

Problem 3 *The Revenue* R *in terms of the number of items produced is given by* $R(x) = 3x$ *and the cost* C *by* $C(x) = 2x + 7$*. Then, the break-even point and the break-even price are*

$$(a) \quad 7, 21 \quad (b) \quad 3, 9 \quad (c) \quad 3, 13 \quad (d) \quad \frac{1}{3}, 1$$

Solution:

We set $R(x) = C(x)$, whence $3x = 2x + 7$, i.e., $x = 7$. $R(7) = 21$. Hence at $x = 7$ items the company breaks even and the break-even price is 21. (a) is the correct answer. ■

Problem 4 *The supply* S *and the demand* D *in terms of the number of items* q *are given by* $S(q) = \frac{1}{2}q + 4$ *and* $D(q) = -\frac{2}{3}q + 18$*, respectively. Then the equilibrium demand and the equilibrium price are*

$$(a) \quad 10, 9 \quad (b) \quad 12, 10 \quad (c) \quad 3, 16 \quad (d) \quad 1, \frac{17}{4}$$

Solution:

We set $S(q) = D(q)$. Then $\frac{1}{2}q + 4 = -\frac{2}{3}q + 18$, which gives $\frac{7}{6}q = 14$, i.e., $q = 12$. Thus, the equilibrium price would be $S(12) = 10$. Hence (b) is the correct answer. ■

Problem 5 *The solutions of* $17x^2 - 17x = 0$ *are*

$$(a) \quad 0, -1 \quad (b) \quad 0, 1 \quad (c) \quad -1, 1 \quad (d) \quad 1, 17$$

Solution:

We have $17x^2 - 17x = 0$, whence $17x(x-1) = 0$, and, therefore $x = 0$ or $x - 1 = 0$, i.e., $x = 0$ or $x = 1$. Thus, (b) is the correct answer. ■

Problem 6 $4x^2 - 8x - 5 = 0$ *has*

$$(a) \quad 0 \quad (b) \quad 1 \quad (c) \quad 2 \quad (d) \quad 3 \text{ solutions}$$

Solution:

We get $D = b^2 - 4ac = (-8)^2 - 4 \cdot 4 \cdot (-5) = 64 + 80 = 144$. Since $D > 0$ the quadratic has 2 different solutions. Thus (c) is the right answer. ■

Problem 7 *George wants to buy a rug for a hallway that is 2 feet by 4 feet. He wants to leave a uniform strip of floor around the rug. Since he is a logician, he can only afford 3 square feet of carpeting. Can you help him out by computing what dimensions the rug should have?*

$$(a) \quad 1.5 \times 3.5 \quad (b) \quad 1.75 \times 3.75 \quad (c) \quad 0.5 \times 2.5 \quad (d) \quad 1 \times 3$$

Solution:

Draw the figure to realize that, if x denotes the width of the uniform strip around the rug, then we must have as dimensions of the rug $(2-2x) \times (4-2x)$. Therefore $(2-2x)(4-2x) = 3$ which gives $8 - 8x - 4x + 4x^2 = 3$, i.e., $4x^2 - 12x + 5 = 0$. Use the quadratic formula to find

$$x_{1,2} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 4 \cdot 5}}{2 \cdot 4} = \frac{12 \pm \sqrt{144 - 80}}{8} = \frac{12 \pm 8}{8} = \frac{20}{8} \text{ or } \frac{4}{8}.$$

Note that $x = \frac{20}{8}$ is not valid because $2x \leq 2$. Hence $x = \frac{1}{2}$ and therefore the rug would be of dimensions $(2 - 2\frac{1}{2}) \times (4 - 2\frac{1}{2})$, i.e., 1×3. Hence (d) is the right answer. ■

Problem 8 $|3x - 2| + 4 > 6$ *has solutions*

$$(a) \quad x \leq 0 \text{ or } x > \frac{4}{3} \quad (b) \quad 0 < x < \frac{4}{3} \quad (c) \quad x < 0 \text{ or } x > \frac{4}{3} \quad (d) \quad 0 \leq x < \frac{4}{3}$$

Solution:

We have $|3x - 2| + 4 > 6$, whence $|3x - 2| > 2$, i.e., $3x - 2 < -2$ or $3x - 2 > 2$, which gives $3x < 0$ or $3x > 4$, which finally results in

$$x < 0 \quad \text{or} \quad x > \frac{4}{3}.$$

Thus (c) is the right answer. ■