HOMEWORK 4: SOLUTIONS - MATH 111
INSTRUCTOR: George Voutsadakis

Problem 1 *Use the quadratic formula to solve* $6x^2 - x - 2 = 0$. *The solutions are*

$$(a) \quad \frac{1}{2}, -\frac{2}{3} \quad (b) \quad \frac{1}{2}, \frac{3}{2} \quad (c) \quad -\frac{1}{2}, \frac{3}{2} \quad (d) \quad -\frac{1}{2}, \frac{2}{3}$$

Solution:
We have

$$D = b^2 - 4ac = (-1)^2 - 4 \cdot 6 \cdot (-2) = 1 + 48 = 49.$$

Hence

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-1) \pm \sqrt{49}}{2 \cdot 6} = \frac{1 \pm 7}{12} = \frac{2}{3} \text{ or } -\frac{1}{2}.$$

Thus (d) is the correct answer. ■

Problem 2 *Use the quadratic formula to solve the equation* $10x^2 - 11x + 3 = 0$. *The solutions are*

$$(a) \quad \textit{no solutions} \quad (b) \quad \frac{3}{5}, \frac{1}{2} \quad (c) \quad \frac{5}{3}, \frac{1}{2} \quad (d) \quad -\frac{3}{5}, -\frac{1}{2}$$

Solution:
We have

$$D = b^2 - 4ac = (-11)^2 - 4 \cdot 10 \cdot 3 = 121 - 120 = 1.$$

Hence

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-11) \pm \sqrt{1}}{2 \cdot 10} = \frac{11 \pm 1}{20} = \frac{3}{5} \text{ or } \frac{1}{2}.$$

Hence (b) is the right answer. ■

Problem 3 *Solve the inequality* $x^2 - 7x + 12 \geq 2$. *The solution is*

$$(a) \quad 2 < x < 5 \quad (b) \quad x < 1 \textit{ or } x > 6 \quad (c) \quad x \leq 1 \textit{ or } x \geq 6 \quad (d) \quad x \leq 2 \textit{ or } x \geq 5$$

Solution:
We have $x^2 - 7x + 12 \geq 2$ implies $x^2 - 7x + 10 \geq 0$, i.e., $(x-2)(x-5) \geq 0$. Now construct the sign table to see that $(x-2)(x-5)$ is positive or zero in the intervals

$$x \leq 2 \quad \text{or} \quad x \geq 5.$$

Hence (d) is the correct answer. ■

Problem 4 *Solve the inequality* $\frac{x-2}{x+3} \leq 0$. *The solutions are*

$$(a) \quad -3 \leq x < 2 \quad (b) \quad -3 \leq x \leq 2 \quad (c) \quad -3 < x \leq 2 \quad (d) \quad x < -3 \textit{ or } x \geq 2$$

Solution:

Form the sign table to get the sign of the product $(x-2)(x+3)$. This is negative or zero in the interval $-3 \leq x \leq 2$. However -3 zeros the denominator of the given fraction and has to be excluded from the solution set. Hence the interval of the solutions is

$$-3 < x \leq 2.$$

Thus (c) is the right answer. ■

Problem 5 *The domain of* $f(x) = |x|$ *is*

$$(a) \quad \mathbb{R} \quad (b) \quad \{x : x \geq 0\} \quad (c) \quad \mathbb{R} - \{0\} \quad (d) \quad \{x : x > 0\}$$

Solution:

No denominators and no square roots appear. So there are no restrictions that have to be taken care of. The answer, thus, is the whole set of real numbers $\mathbb{R}$. I.e., (a) is the correct answer. ■

Problem 6 *The domain of* $g(x) = \sqrt{\frac{x^2-2x+1}{x-3}}$ *is*

$$(a) \quad \{x : x \leq 1 \text{ or } x > 3\} \quad (b) \quad \{x : x \geq 3\} \quad (c) \quad \{x : x > 3\} \quad (d) \quad \mathbb{R} - \{3\}$$

Solution:

We must take the two restrictions

1. $x - 3 \neq 0$ and
2. $\frac{x^2-2x+1}{x-3} \geq 0$.

The first gives $x \neq 3$. The second, after setting up the sign table, yields $x = 1$ or $x > 3$. Thus $\{x : x = 1$ or $x > 3\}$ is the right answer. **Unfortunately, because of my mistake, this correct answer does not appear in the multiple choice answers...** ■

Problem 7 *Graph the piece-wise linear function*

$$f(x) = \begin{cases} -x-2, & \text{if } x \leq 2 \\ 2x+3, & \text{if } x > 2 \end{cases}$$

Solution:

Draw the graphs of $g(x) = -x - 2$ for $x \leq 2$ and of $h(x) = 2x + 3$ for $x > 2$ on the same coordinate axes. You would obtain the following graph

■

Problem 8 *Consider the function $g(x) = -x^2 + 8x - 15$. Its graph is a parabola. Find its vertex and x-intercepts, state whether it opens up or down and make a rough sketch of it.*

Solution:

The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{8}{2\cdot(-1)} = 4$. Thus, the y-coordinate is $f(4) = -4^2 + 8 \cdot 4 - 15 = -16 + 32 - 15 = 1$. Thus $V = (4, 1)$.

The x-intercepts may be found by setting $y = 0$. Then $-x^2 + 8x - 15 = 0$, i.e., $x^2 - 8x + 15 = 0$. This gives $(x - 3)(x - 5) = 0$, whence $x = 3$ or $x = 5$. The x-intercepts are therefore the points $(3, 0)$ and $(5, 0)$.

Since $a = -1 < 0$, the parabola opens down.

Its rough sketch follows:

■