EXAM 1: SOLUTIONS - MATH 111
INSTRUCTOR: George Voutsadakis

Problem 1 *Find the equation of the line that is perpendicular to* $2x - 7y = 21$ *and passes through the point* $(-2, 7)$.

Solution:
The slope of the given line may be found by solving its equation for y: We have: $7y = 2x - 21$, whence $y = \frac{2}{7}x - 3$. Hence $m = \frac{2}{7}$. The unknown line is perpendicular to the given one, whence its slope is also $-\frac{7}{2}$. Since, it goes through the point $(-2, 7)$, its equation is given by the point-slope form:

$$y - 7 = -\frac{7}{2}(x - (-2)), \quad \text{i.e.,} \quad y - 7 = -\frac{7}{2}x - 7,$$

or $y = -\frac{7}{2}x$. ■

Problem 2 *Find the equation of the line that goes through the points* $(-4, 3)$ *and* $(5, 27)$.

Solution:
The slope of the line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{27 - 3}{5 - (-4)} = \frac{24}{9} = \frac{8}{3}.$$

Thus, its equation is $y - 3 = \frac{8}{3}(x - (-4))$ i.e., $y - 3 = \frac{8}{3}x + \frac{32}{3}$ or $y = \frac{8}{3}x + \frac{41}{3}$. ■

Problem 3 *The cost* C *in terms of the number of items* x *produced is given by* $C(x) = 7x + 36$ *and the revenue by* $R(x) = 9x$. *Find the range of values of* x *for which the company will at least break even and the revenue, when the company breaks even.*

Solution:
The company will at least break even when $R(x) \geq C(x)$. Thus we have $9x \geq 7x + 36$, which gives $2x \geq 36$, i.e., $x \geq 18$.
The revenue when the company breaks even is given by $R(18) = 9 \cdot 18 = 162$. ■

Problem 4 *The demand price* p *of an item in terms of the quantity* q *is given by* $p = -q^2 + 120$ *and the supply price* p *in term of the quantity* q *by* $p = 10q$. *Determine the equilibrium price and the equilibrium supply.*

Solution:
To find the equilibrium quantity (both supply and demand) we set $-q^2 + 120 = 10q$, whence $q^2 + 10q - 120 = 0$. We use the quadratic formula to solve for q. The discriminant is

$$D = b^2 - 4ac = 10^2 - 4 \cdot 1 \cdot (-120) = 580.$$

Therefore

$$q_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-10 \pm \sqrt{580}}{2} = \frac{-10 \pm 2\sqrt{145}}{2} = -5 \pm \sqrt{145}$$

and, since the quantity cannot be negative we get $q = -5 + \sqrt{145}$. Now the equilibrium price is given by $p = 10(-5 + \sqrt{145})$. ■

Problem 5 *Solve the inequality* $|2x - \frac{7}{3}| - 8 \leq 15$.

Solution:
We have $|2x - \frac{7}{3}| - 8 \leq 15$ implies $|2x - \frac{7}{3}| \leq 23$, whence $-23 \leq 2x - \frac{7}{3} \leq 23$, and, therefore, $-23 + \frac{7}{3} \leq 2x \leq 23 + \frac{7}{3}$, i.e., $-\frac{69}{3} + \frac{7}{3} \leq 2x \leq \frac{69}{3} + \frac{7}{3}$, whence $-\frac{62}{3} \leq 2x \leq \frac{76}{3}$. Therefore $-\frac{31}{3} \leq x \leq \frac{38}{3}$. ■

Problem 6 *Find the domain of* $f(x) = \sqrt{\frac{x-3}{-x+7}}$.

Solution:
First $-x + 7 \neq 0$, whence $x \neq 7$. Since $\frac{x-3}{-x+7}$ appears underneath a square root, we must also have $\frac{x-3}{-x+7} \geq 0$. We may now set up the sign table to find $3 \leq x < 7$. ■