EXAM 2: SOLUTIONS - MATH 111
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Problem 1 *Find the vertex, the opening direction, the x- and y-intercepts and sketch the graph of* $f(x) = \frac{1}{3}x^2 + \frac{2}{3}x - 1$.

Solution:

The vertex has x-coordinate

$$x = -\frac{b}{2a} = -\frac{\frac{2}{3}}{2 \cdot \frac{1}{3}} = -1$$

and y-coordinate $f(-1) = \frac{1}{3} - \frac{2}{3} - 1 = -\frac{4}{3}$. Hence, it is the point $(-1, -\frac{4}{3})$.

The parabola opens up, since $a = \frac{1}{3} > 0$.

The y-intercept is found by setting $x = 0$. We have then $y = -1$. Thus $(0, -1)$ is the y-intercept. The x-intercepts are found by setting $y = 0$ and solving $\frac{1}{3}x^2 + \frac{2}{3}x - 1 = 0$. We have $x^2 + 2x - 3 = 0$, i.e., $(x + 3)(x - 1) = 0$, whence $x = -3$ or $x = 1$. Thus, the x-intercepts are the points $(-3, 0)$ and $(1, 0)$.

The graph follows

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Problem 2 *Find the equation of the parabola that has vertex* $V = (2, -3)$ *and goes through the point* $(-1, -5)$.

Solution:

Since the vertex is at $V = (2, -3)$, we have equation

$$f(x) = a(x - 2)^2 - 3.$$

But the parabola goes through $(-1, -5)$, whence

$$-5 = a(-1 - 2)^2 - 3, \quad \text{i.e.,} \quad -5 = 9a - 3,$$

which yields $a = -\frac{2}{9}$. Thus, the equation of the parabola is

$$f(x) = -\frac{2}{9}(x - 2)^2 - 3.$$

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Problem 3 *The demand for a certain type of cosmetic is given by $p = 500 - x$, where p is the price when x units are demanded. Find an expression for the revenue in terms of the number x of units. Then graph the revenue function and find the maximum revenue.*

Solution:

The revenue is the number of units sold times the price per unit. Thus $R(x) = xp(x) = x(500 - x) = -x^2 + 500x$. For the graph, find the vertex of this parabola ($V = (250, 62500)$) and the x intercepts ($x = 0, x = 500$) and then plot the graph. Obviously, the maximum revenue is $R_{\max} = 62500$ and occurs when $x = 250$ units are sold.

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Problem 4 *Create the sign table and make a rough sketch of the graph of the function $f(x) = x^4 - 4x^2$.*

Solution:

We have $f(x) = x^4 - 4x^2 = x^2(x^2 - 4) = x^2(x + 2)(x - 2)$. Thus the roots are $-2, 0$ and 2. Using these three values we set the sign table and roughly plot the graph of the function.

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Problem 5 *Find the domain, the horizontal and vertical asymptotes and roughly sketch the graph of the function $g(x) = \frac{-3x+5}{x+2}$.*

Solution:

The domain should exclude the values that zero the denominator. Thus, in this case $D(g) = \mathrm{I\!R} - \{-2\}$. This also gives the vertical asymptote which is the line $x = -2$. For the horizontal asymptote we divide the coefficient of the first degree term in the numerator by the coefficient of the first degree term of the denominator and get the line $y = \frac{-3}{1} = -3$. Now using this information and computing the x- and y-intercepts, which are $(\frac{5}{3}, 0)$ and $(0, \frac{5}{2})$, respectively, we can plot the graph of g:

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Problem 6 *Solve the exponential equation* $2003^{3x^2-7x} = 1$.

Solution:

We have $2003^{3x^2-7x} = 1$ implies $2003^{3x^2-7x} = 2003^0$, whence $3x^2 - 7x = 0$, i.e., $x(3x - 7) = 0$. Therefore, we obtain the two roots $x = 0$ or $x = \frac{7}{3}$. ■