HOMEWORK 5: SOLUTIONS - MATH 111
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Problem 1 *Create the sign table and graph the function* $f(x) = x^4 - 6x^2 + 8$.

Solution:

We have $f(x) = 0$ implies $x^4 - 6x^2 + 8 = 0$, whence $(x^2 - 2)(x^2 - 4) = 0$. Thus $(x - \sqrt{2})(x + \sqrt{2})(x - 2)(x + 2) = 0$, and therefore the sign table must have four points $-2, -\sqrt{2}, \sqrt{2}$ and 2 and five rows corresponding to $x - \sqrt{2}, x + \sqrt{2}, x - 2, x + 2$ and $f(x)$. The sign of $f(x)$ turns out to be $+$ if $x < -2$, $-$ if $-2 < x < -\sqrt{2}$, $+$ if $-\sqrt{2} < x < \sqrt{2}$, $-$ if $-\sqrt{2} < x < 2$ and, finally $+$ if $x > 2$.

The rough sketch follows:

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Problem 2 *Study the function* $f(x) = \frac{2x-7}{-5x+2}$. *(***Studying** *here means what we did in class for rational functions: Find the domain, find the x- and y-intercepts, find the horizontal and vertical asymptotes and then roughly plot the graph.)*

Solution:

The domain is $D(f) = \mathbb{R} - \{\frac{2}{5}\}$, since $\frac{2}{5}$ is a root of the denominator. For the x-intercept, set $y = 0$. Then $2x - 7 = 0$, whence $x = \frac{7}{2}$. Thus $(\frac{7}{2}, 0)$ is the x-intercept. For the y-intercept, set $x = 0$. We get $y = -\frac{7}{2}$, i.e., $(0, -\frac{7}{2})$ is the y-intercept. The horizontal asymptote is $y = \frac{2}{-5} = -\frac{2}{5}$ and the vertical asymptote is $x = \frac{2}{5}$.

The rough sketch follows:

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Problem 3 *Find the equations of the vertical and horizontal asymptotes of the function* $f(x) = \frac{x^2-4x+3}{x^2-6x+5}$.

Solution:

The horizontal asymptote is $y = \frac{1}{1} = 1$. To find the vertical asymptotes, we factor both numerator and denominator into linear factors: $f(x) = \frac{(x-3)(x-1)}{(x-5)(x-1)}$. Thus, the only vertical asymptote occurs at the root $x = 5$ of the denominator that is not a root of the numerator.

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Problem 4 *Graph on the same axes the functions* $f(x) = 3^x, g(x) = 3^{-x}$ *and* $h(x) = -3^x$. *Before graphing compute their values at* $x = -1, x = 0$ *and* $x = 1$ *and depict those* clearly *both on a small table* and *on your graphs.*

Solution:

We have

x	$f(x)$	$g(x)$	$h(x)$
-1	$\frac{1}{3}$	3	$-\frac{1}{3}$
0	1	1	-1
1	3	$\frac{1}{3}$	-3

The graphs follow:

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Problem 5 *Solve the equation* $9^{x^2} = 81^{\frac{3}{2}x+5}$.

Solution:

We have $9^{x^2} = 81^{\frac{3}{2}x+5}$ implies $9^{x^2} = (9^2)^{\frac{3}{2}x+5}$, whence $x^2 = 2(\frac{3}{2}x + 5)$. Therefore $x^2 = 3x + 10$, which yields $x^2 - 3x - 10 = 0$. This gives $(x - 5)(x + 2) = 0$, i.e., $x = -2$ or $x = 5$. ■

Problem 6 *Solve the equation* $7^{3x+7} = (\frac{1}{7})^{5x-8}$.

Solution:

We have $7^{3x+7} = (\frac{1}{7})^{5x-8}$ implies $7^{3x+7} = (7^{-1})^{5x-8}$, whence $3x + 7 = -(5x - 8)$. Therefore $3x + 7 = -5x + 8$. Hence $8x = 1$, which, finally, yields $x = \frac{1}{8}$. ■

Problem 7 *Culture studies in the lab have determined that the population of an organism A as a function of time t is given by $f(t) = e^{t^2}$. At the same time, the population of another organism B in the same culture has been increasing according to the function $g(t) = \sqrt{e}^{16t+40}$. At what time will the two organisms have the same populations in the culture?*

Solution:

We have $e^{t^2} = \sqrt{e}^{16t+40}$, whence $e^{t^2} = (e^{\frac{1}{2}})^{16t+40}$, i.e., $t^2 = \frac{1}{2}(16t + 40)$. This gives $t^2 = 8t + 20$, whence $t^2 - 8t - 20 = 0$. We thus get $(t - 10)(t + 2) = 0$, i.e., $t = -2$ or $t = 10$. Since t represents time $t = 10$. ■

Problem 8 *Compute* $\ln(\sqrt[5]{e^{15}})$ *and* $\ln(e^{\frac{2}{5}})$ *without using a calculator.*

Solution:

We get $\ln(\sqrt[5]{e^{15}}) = \ln e^{\frac{15}{5}} = \frac{15}{5} = 3$. Similarly, $\ln(e^{\frac{2}{5}}) = \frac{2}{5}$. ■