HOMEWORK 6: SOLUTIONS - MATH 111 INSTRUCTOR: George Voutsadakis

Problem 1 If $\ln x = 7$ and $\ln y = 2$ find $\ln \left(\frac{\sqrt{x^3}}{y^5}\right)$.

Solution:

We have

$$\begin{split} \ln\left(\frac{\sqrt{x^3}}{y^5}\right) &= & \ln\sqrt{x^3} - \ln y^5 \\ &= & \ln\left(x^{\frac{3}{2}}\right) - \ln\left(y^5\right) \\ &= & \frac{3}{2}\ln x - 5\ln y \\ &= & \frac{3}{2}7 - 5\cdot 2 \\ &= & \frac{1}{2}. \end{split}$$

Problem 2 Solve the equation $\log_3(x-3) - \log_3(x-5) = 2$.

Solution:

 $\log_3(x-3) - \log_3(x-5) = 2$ implies $\log_3 \frac{x-3}{x-5} = 2$ which gives $\frac{x-3}{x-5} = 3^2$, i.e., x-3 = 9(x-5). This yields x-3 = 9x-45, whence 8x = 42, and, therefore, $x = \frac{42}{8} = \frac{21}{4}$. Note that since both $\frac{21}{4} - 3 > 0$ and $\frac{21}{4} - 5 > 0$, this is an admissible solution.

Problem 3 Solve the equation $\log_{119}(x+2) + \log_{119}(x-8) = 1$.

Solution:

We have $\log_{119}(x+2) + \log_{119}(x-8) = 1$ implies $\log_{119}(x+2)(x-8) = 1$, whence $(x+2)(x-8) = 119^1$, i.e., $x^2 - 6x - 16 = 119$ or $x^2 - 6x - 135 = 0$. This has the factorization (x-15)(x+9) = 0, whence x = -9 or x = 15. However, since the two logarithms $\log_{119}(x+1)$ and $\log_{119}(x-9)$ are not allowed to have negative arguments, x = -9 is not an admissible solution. Only x = 15 is admissible!

Problem 4 Solve the equation $\log (x^{16}) = (\log x)^2$.

Solution:

 $\log (x^{16}) = (\log x)^2$ gives $16 \log x = (\log x)^2$, whence $(\log x)^2 - 16 \log x = 0$. Hence $\log x (\log x - 16) = 0$. Therefore $\log x = 0$ or $\log x = 16$. Thus the solutions for x are $x = 10^0$ or $x = 10^{16}$, i.e., x = 1 or $x = 10^{16}$.

Problem 5 In the central Sierra Nevada of California, the percent of moisture that falls as snow rather than rain is approximated reasonably well by $p = 86.3 \ln h - 680$, where p is the percent of moisture as snow at an altitude of h feet (with $3000 \le h < 8500$).

- (a) Graph p.
- (b) At what altitude is 50 percent of the moisture snow?

Solution:

(a) You should first graph $\ln h$, then graph $86.3 \ln h$ (which is a stretching of $\ln h$ and, finally, graph p, which is a vertical shift of $86.3 \ln h$ by 680 units south!

(b) For this part, we are seeking x such that p = 50. We have $86.3 \ln h - 680 = 50$ implies $86.3 \ln h = 730$, whence $\ln h = \frac{730}{86.3}$, i.e., $h = e^{\frac{730}{86.3}} \approx 4716.7$. Thus, the altitude will be approximately 4716.7 ft.

Problem 6 Find the simple interest on a loan of \$10,000 at 3% made on September 1 and due on November 30.

Solution:

We have $I = Prt = 10000 \cdot 0.03 \cdot \frac{3}{12} = 75.$

Problem 7 A friend of yours decided to go back to college. She decides to buy a small car for \$9,000. She intends to borrow the money from a bank with 10% discount rate. If she plans to repay the loan in 5 years what will be the amount of her loan?

Solution:

We have P = A(1 - rt), whence $A = \frac{P}{1 - rt} = \frac{9000}{1 - 0.1 \cdot 5} = \frac{9000}{0.5} = 18,000.$

Problem 8 Find the amount of interest earned by a deposit of \$5,000 compounded quarterly at 5% for 10 years.

Solution: $I = A - P = P(1 + \frac{r}{m})^{mt} - P = 5000(1 + \frac{0.05}{4})^{4 \cdot 10} - 5000 = 5000(1.0125)^{40} - 5000 \approx 3,218.$