# HOMEWORK 7: SOLUTIONS - MATH 111
# INSTRUCTOR: George Voutsadakis

**Problem 1** *Find the present value of the future amount \$8,000 compounded quarterly at 4% for 10 years.*

**Solution:**

We have $A = P(1+\frac{r}{m})^{mt}$, i.e., $P = \frac{A}{(1+\frac{r}{m})^{mt}}$, whence $P = \frac{8000}{(1+\frac{0.04}{4})^{4\cdot 10}}$ and, therefore, $P = \frac{8000}{1.01^{40}}$. ■

**Problem 2** *Find the sum of the first five terms of the geometric sequence with first term $a = 7$ and common ratio $r = 3$.*

**Solution:**

$S_5 = a\frac{r^n-1}{r-1} = 7\frac{3^5-1}{3-1} = 7\frac{242}{2} = 7 \cdot 121 = 847.$ ■

**Problem 3** *Solve the systems*

$$\left\{ \begin{array}{rcrcr} 3x & + & 8y & = & -18 \\ -x & + & 3y & = & -11 \end{array} \right\}, \quad \left\{ \begin{array}{rcrcr} 5x & - & y & = & 2 \\ -10x & + & 2y & = & -3 \end{array} \right\},$$

*by the* **substitution method**.

**Solution:**

(a) We have $-x + 3y = -11$ implies $x = 3y + 11$, and, therefore, $3(3y + 11) + 8y = -18$. Hence $9y + 33 + 8y = -18$, which yields $17y = -51$, i.e., $y = -3$. Back substituting, we get $x = 2$. Therefore the solution pair is $(x, y) = (2, -3)$.

(b) $5x - y = 2$ gives $y = 5x - 2$, whence $-10x + 10x - 4 = -3$, which gives $-4 = -3$. Hence the given system has no solutions. ■

**Problem 4** *Solve the system* $\left\{ \begin{array}{rcrcrcr} -x & + & 2y & - & z & = & -12 \\ 2x & - & y & + & 3z & = & 20 \\ x & - & 3y & + & 2z & = & 20 \end{array} \right\}$ *by the* **elimination method**.

**Solution:**

The first and third equations give $-y + z = 8$, i.e., $z = y + 8$. The first and second equations give $3y + z = -4$. Therefore $3y + y + 8 = -4$, whence $y = -3$. Thus $z = y + 8 = 5$. Now any of the three equations gives $x = 1$. Therefore the triple $(x, y, z) = (1, -3, 5)$ is the unique solution triple. ■

**Problem 5** *Solve the system* $\left\{ \begin{array}{rcrcrcr} -5x & + & y & - & z & = & 12 \\ -10x & + & 3y & + & z & = & 11 \\ -7x & - & 2y & + & 2z & = & -7 \end{array} \right\}$ *by using the* **Gauss-Jordan method** *(matrix row operations).*

**Solution:**

$$\left[\begin{array}{rrr|r} -5 & 1 & -1 & 12 \\ -10 & 3 & 1 & 11 \\ -7 & -2 & 2 & 7 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} -5 & 1 & -1 & 12 \\ -7 & -2 & 2 & 7 \\ -10 & 3 & 1 & 11 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} -15 & 4 & 0 & 23 \\ 13 & -8 & 0 & -29 \\ -10 & 3 & 1 & 11 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{rrr|r} -2 & -4 & 0 & -6 \\ 13 & -8 & 0 & -29 \\ -10 & 3 & 1 & 11 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 2 & 0 & 3 \\ 13 & -8 & 0 & -29 \\ -10 & 3 & 1 & 11 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 2 & 0 & 3 \\ 0 & -34 & 0 & -68 \\ 0 & 23 & 1 & 41 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{rrr|r} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 23 & 1 & 41 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -5 \end{array}\right].$$

Thus, the solution is $(x, y, z) = (-1, 2, -5)$. ■

**Problem 6** *Solve the system* $\left\{\begin{array}{rcrcrcr} 2x & - & 6y & + & z & = & 30 \\ -x & + & y & + & 2z & = & 21 \\ x & - & 3y & + & 3z & = & 45 \end{array}\right\}$ *by using the* **Gauss-Jordan method** *(matrix row operations).*

**Solution:**

$$\left[\begin{array}{rrr|r} 2 & -6 & 1 & 30 \\ -1 & 1 & 2 & 21 \\ 1 & -3 & 3 & 45 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & -3 & 3 & 45 \\ -1 & 1 & 2 & 21 \\ 2 & -6 & 1 & 30 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & -3 & 3 & 45 \\ 0 & -2 & 5 & 66 \\ 0 & 0 & -5 & -60 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{rrr|r} 1 & -3 & 3 & 45 \\ 0 & 1 & -\frac{5}{2} & -33 \\ 0 & 0 & 1 & 12 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 0 & -\frac{9}{2} & -54 \\ 0 & 1 & -\frac{5}{2} & -33 \\ 0 & 0 & 1 & 12 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 12 \end{array}\right].$$

Thus, we have the solution $(x, y, z) = (0, -3, 12)$. ■

**Problem 7** *Let* $A = \begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$. *Compute* $A+B$, $A-B$ *and* $3A-2B$.

**Solution:**
We have

$$A + B = \begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 11 & -4 \end{bmatrix}.$$

$$A - B = \begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 10 \end{bmatrix}.$$

$$3A - 2B = 3\begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix} - 2\begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -14 \\ 15 & -21 \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ 12 & 6 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ 3 & -27 \end{bmatrix}.$$

■

**Problem 8** *Let* $A = \begin{bmatrix} -1 & 7 & -5 \\ 2 & -3 & 1 \end{bmatrix}$ *and* $B = \begin{bmatrix} -3 & -5 & 9 \\ 2 & 11 & -7 \end{bmatrix}$. *Compute* $A - B$ *and* $-2A + 5B$.

**Solution:**

$$A - B = \begin{bmatrix} -1 & 7 & -5 \\ 2 & -3 & 1 \end{bmatrix} - \begin{bmatrix} -3 & -5 & 9 \\ 2 & 11 & -7 \end{bmatrix} = \begin{bmatrix} 2 & 12 & -14 \\ 0 & -14 & 8 \end{bmatrix}.$$

$$-2A + 5B = -2 \begin{bmatrix} -1 & 7 & -5 \\ 2 & -3 & 1 \end{bmatrix} + 5 \begin{bmatrix} -3 & -5 & 9 \\ 2 & 11 & -7 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & -14 & 10 \\ -4 & 6 & -2 \end{bmatrix} + \begin{bmatrix} -15 & -25 & 45 \\ 10 & 55 & -35 \end{bmatrix} = \begin{bmatrix} -13 & -39 & 55 \\ 6 & 61 & -37 \end{bmatrix}.$$

■