EXAM 1 - MATH 215

Friday, October 3, 2003
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 4 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Let P and Q be propositional forms. Define the *new connective* $\Box$ by the following truth table

P	Q	$P \Box Q$
F	F	F
F	T	T
T	F	T
T	T	F

   (a) Prove that $P \Box Q$ is equivalent to the negation of $P \Leftrightarrow Q$.

   (b) Is $P \Box Q$ equivalent to $(P \vee Q) \wedge \sim (P \wedge Q)$?

2. (a) Give the definition of the converse and of the contrapositive of an implication.

   (b) Prove that the contrapositive is equivalent to the original implication.

3. (a) Is $((P \wedge Q) \Rightarrow R) \Rightarrow ((P \Rightarrow R) \wedge (Q \Rightarrow R))$ a tautology? Explain!

   (b) Prove that in $\mathbb{N}$,

$$(\exists x)(x + 3 = -x + 8) \Leftrightarrow (\forall x)(x^2 + 5 \text{ is prime}).$$

4. (a) Suppose that a, b, c are positive integers. Prove that if a divides b and a divides $b + c$, then a divides c.

   (b) Prove by contradiction that there is no odd integer that can be simultaneously expressed in the forms $4j - 1$ and $4k + 1$ for integers j and k.

5. (a) Prove that if $x < 1$ or $x > 3$, then $\frac{x-1}{x-3} > 0$.

   (b) Prove or disprove the following quantified statement: There is a unique three-digit number whose digits have sum 8 and product 10.