HOMEWORK 1 - MATH 215

DUE DATE: When Section 1.3 has been covered!
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Make truth tables for the propositional forms $(P \wedge Q) \vee (P \wedge R)$ and $(P \wedge Q) \vee (R \wedge \sim S)$.

2. Which of the following pairs of propositional forms are equivalent?
   (a) $\sim (P \wedge Q), \sim P \wedge \sim Q$ (b) $\sim (P \vee Q), (\sim P) \wedge (\sim Q)$.

3. If P, Q and R are true while S and T are false, which of the following are true?
   (a) $((\sim P) \vee (\sim Q)) \vee ((\sim R) \vee (\sim S))$ (b) $((\sim T) \wedge P) \vee (T \wedge P)$.

4. Give a useful denial of the following statements:
   (a) We will win the first game or the second one.
   (b) n is even and n is not a multiple of 5.

5. P, Q and R are propositional forms, P is equivalent to Q and Q is equivalent to R. Prove that $\sim Q$ is equivalent to $\sim P$ and that $P \wedge Q$ is equivalent to $Q \wedge R$.

6. Determine whether each of the following is a tautology, a contradiction or neither. Prove your answers.
   (a) $(P \wedge Q) \vee ((\sim P) \wedge (\sim Q))$ (b) $(Q \wedge \sim P) \wedge \sim (P \wedge R)$.

7. Which of the following conditional sentences are true?
   (a) If a hexagon has six sides, then the moon is made of cheese.
   (b) If Euclid's birthday was April 2, then rectangles have 4 sides.
   (c) $1 + 1 = 2$ is sufficient for $3 > 6$
   (d) Horses have 4 legs whenever September 15 falls on a Saturday.

8. Which of the following are true?
   (a) $7 + 5 = 12$ iff $1 + 1 = 2$
   (b) A parallelogram has three sides iff 27 is a prime
   (c) The Eiffel Tower is in Paris iff the chemical symbol for helium is H
   (d) $\sqrt{10} + \sqrt{13} < \sqrt{11} + \sqrt{12}$ iff $\sqrt{13} - \sqrt{12} < \sqrt{11} - \sqrt{10}$.

9. Make truth tables for the propositional forms $(P \wedge Q) \vee (Q \wedge R) \Rightarrow P \vee R$ and $[(Q \Rightarrow S) \wedge (Q \Rightarrow R)] \Rightarrow [(P \vee Q) \Rightarrow (S \vee R)]$.

10. Show that the following pairs of statements are equivalent:
(a) $P \Rightarrow (Q \vee R)$ and $(P \wedge \sim R) \Rightarrow Q$
(b) $(P \Rightarrow Q) \Rightarrow R$ and $(P \wedge \sim Q) \vee R$
(c) $P \Leftrightarrow Q$ and $(\sim P \vee Q) \wedge (\sim Q \vee P)$.

11. Give, if possible, an example of a true conditional sentence for which (a) the converse is false (b) the contrapositive is true.

12. Give, if possible, an example of a false conditional sentence for which (a) the converse is true (b) the contrapositive is true.

13. Determine whether each of the following is a tautology, a contradiction or neither:
(a) $(P \Leftrightarrow Q) \Leftrightarrow \sim (\sim P \vee Q) \vee (\sim P \wedge Q)$
(b) $(P \vee Q) \Rightarrow Q \Rightarrow P$
(c) $[P \Rightarrow (Q \wedge R)] \Rightarrow [R \Rightarrow (P \Rightarrow Q)]$.

14. Translate the following English sentences into symbolic sentences with quantifiers. The universe for each is given in parentheses.
(a) No right triangle is isosceles. (All triangles)
(b) All people are honest or no one is honest. (All people)
(c) For every positive real number x, there is a unique real number y such that $2^y = x$. (Real numbers)

15. Which of the following are true? The universe for each is given in parentheses.
(a) $(\forall x)(x + x \geq x)$ (Real numbers)
(b) $(\exists x)(2x + 3 = 6x + 7)$ (Natural numbers)
(c) $(\exists x)(3(2 - x) = 5 + 8(1 - x))$ (Real numbers)
(d) $(\forall x)(x^2 + 6x + 5 \geq 0)$ (Real numbers)

16. Give an English translation for each. The universe is given in parentheses.
(a) $(\forall x)(x$ is prime $\wedge\, x \neq 2 \Rightarrow x$ is odd) (Natural numbers)
(b) $\sim (\exists x)(x^2 < 0)$ (Real numbers)
(c) $(\forall x)(x$ is odd $\Rightarrow x^2$ is odd) (Natural numbers)

17. Which of the following are true for the universe of all real numbers?
(a) $(\exists x)(\forall y)(x + y = 0)$
(b) $(\exists x)(\exists y)(x^2 + y^2 = -1)$
(c) $(\forall y)(\exists! x)(x = y^2)$
(d) $(\exists! x)(\exists! y)(\forall w)(w^2 > x - y)$

18. Give a proof of $(\exists! x)P(x) \Rightarrow (\exists x)P(x)$. Then show that the converse of this conditional sentence is false.