EXAM 1 - MATH 325

Thursday, October 9, 2003
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Read each problem very carefully before starting to solve it. Each question is worth 4 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) State and prove the Vertical Angle Theorem.

   (b) Consider a triangle $\triangle ABC$. Prove that $\overline{AB} \cong \overline{AC}$ if and only if $\widehat{B} \cong \widehat{C}$.

2. (a) Define the notion of *congruence* for triangles. Then state (carefully) the SSS Criterion for congruent triangles.

   (b) Given a line l and a point P not on l, construct the perpendicular to l through P. Prove that your construction works.

3. (a) State and prove the Triangle Inequality Theorem.

   (b) Define the notion of *parallelism* between lines. Then show that through a given point P outside a given line l, there passes at least one parallel to l.

4. (a) Let $ABCD$ be a quadrilateral. Show that $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ if and only if $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$.

   (b) State and prove the converse to the Pythagorean Theorem.

5. (a) Assume the following Proposition:

   Proposition: Given two triangles $\triangle ABC$ and $\triangle DEF$, such that $\widehat{A} \cong \widehat{D}$ and $\overline{AB} \cong \overline{DE}$, then $\frac{\text{Area}(ABC)}{\text{Area}(DEF)} = \frac{AC}{DF}$.

   Use this proposition to show that, given two triangles $\triangle ABC$ and $\triangle DEF$, such that $\widehat{A} \cong \widehat{D}$,

   $$\frac{\text{Area}(ABC)}{\text{Area}(DEF)} = \frac{AB \cdot AC}{DE \cdot DF}.$$

   (b) State the Geometric Law of Cosines (one of the two possible formulations suffices). Then use it to compute the length of the height h_a of a given triangle in terms of the lengths a, b, c of its sides $\overline{BC}, \overline{AC}, \overline{AB}$, respectively.