HOMEWORK 5 - MATH 325

DUE DATE: After Chapter 7 has been covered!
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Prove: if s is the length of a side of an n-gon inscribed in a circle of radius r and t the length of a side of a $2n$-gon inscribed in a circle of radius r, then $t^2 = 2r(r - \sqrt{r^2 - \frac{1}{4}s^2})$.
2. Use the previous exercise to get a lower bound for C based on the regular 24-gon; the regular 48-gon.
3. Assume that $\triangle ABC$ and $\triangle DEF$ are similar with ratio k. Prove that each of the circumradius, the inradius and the exradii of $\triangle DEF$ are k times the corresponding parts of $\triangle ABC$.
4. In each part determine the missing information about $\triangle ABC$.
   (a) $a = 4, b = 4, c = 2, K = ?, R = ?, r = ?, r_a = ?, r_b = ?, r_c = ?$
   (b) $r_a = 2, r_b = 3, r_c = 6, r = ?, K = ?, R = ?, a = ?, b = ?, c = ?$
5. Prove that if a quadrilateral $ABCD$ is circumscribed about a circle, then the area of $ABCD$ is one-half times the radius of the circle times the perimeter.
6. Let the lengths of the three altitudes be h_a, h_b and h_c. Prove that $\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r}$.
7. Prove that the circle with diameter $\overline{I_bI_c}$ has center on the circumcircle and contains the points B and C.
8. Prove $OI_a^2 = R(R + 2r_a)$.