HOMEWORK 1 - MATH 351

DUE DATE: When Chapter 1 has been covered!
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Prove each of the following using mathematical induction: (a) $2+5+8+\ldots+(3n-1)=\frac{n(3n+1)}{2}$.
   (b) $1+t+t^2+\ldots+t^{n-1}=\frac{1-t^n}{1-t}$, where $t\neq 1$.
   (c) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$.
   (d) $n^2-2n\geq n+3$ for $n\geq 4$.
   (e) 7^n-2^n is exactly divisible by 5 for $n\geq 1$.

2. An ice cream parlor serves a sundae for which you can choose one of 20 different flavors, with one of seven different toppings. You can then choose to have whipped cream or not. How many sundaes are possible?

3. a witness to a hit-and-run accident tried to memorize the license plate of the car. She remembers that there were two letters followed by three numbers. The first letter was W and the second letter was either C,D,O or Q. The last two numbers were both 7. How many different plate numbers would the police have to check given this information?

4. A certain large city now has five area codes. a phone number consists of the area code followed by seven digits, the first of which cannot be zero or one. How many phone numbers are possible for that city?

5. How many six-letter strings from $\{a,b,c,\ldots,z\}$ contain exactly one of the vowels a,e,i,o,u (other letters may be repeated)?

6. determine the number of license plate codes consisting of three letters followed by three digits that contain a repeated letter or a repeated digit (or both).

7. There are eight democrats and four Republicans on a senate committee. They must form a subcommittee of five members for a project. How many possible subcommittees are there containing
   (a) exactly two Democrats? (b) at least one republican?

8. A jar contains 10 red, 12 white and 13 blue balls, all of which are distinct. In how many ways can seven balls be selected so that there are at least two balls of each color?

9. (a) How many ordered five-letter sequences can be made using the letters A,A,B,C and D? An example of an ordered sequence using these letters is $BADAC$.
   (b) Generalize part (a) to the case in which you are given n symbols and k of them are the same while the rest are all distinct. How many ordered n-letter sequences are there?

10. Prove the following identity: $1\times 3\times 5\times\ldots\times(2n-1)=\frac{(2n)!}{n!2^n}$.

11. Prove by mathematical induction: $\sum_{i=1}^{n} i(i!) = (n+1)! - 1$.
12. Prove by a combinatorial argument that $\binom{n}{r} = \binom{n-2}{r} + 2\binom{n-2}{r-1} + \binom{n-2}{r-2}$.
13. Find appropriate choices for a and b in the binomial theorem in order to evaluate the given quantity as a function of n:
(a) $\sum_{k=0}^{n} \binom{n}{k} 5^k$ (b) $\sum_{k=0}^{n} \binom{n}{k} 10^{n-k}$.