HOMEWORK 5 - MATH 152

DUE DATE: Monday, November 1
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. One part of each homework problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Evaluate the integrals that converge:
   (a) $\int_2^{+\infty} \frac{1}{x\sqrt{\ln x}} dx$
   (b) $\int_{-\infty}^{+\infty} \frac{x}{(x^2+3)^2} dx$
   (c) $\int_0^8 \frac{1}{\sqrt[3]{x}} dx$
2. Find the area of the region between the x-axis and the curve $y = \frac{8}{x^2-4}$ for $x \geq 3$.
3. The following first-order linear equations can be rewritten as first-order separable equations. Solve the equations using both the method of integrating factors and the method of separation of variables, and determine whether the solutions produced are the same:
   (a) $\frac{dy}{dx} - 4xy = 0$ (b) $\frac{dy}{dt} + y = 0$
4. Solve the differential equations by the method of integrating factors:
   (a) $2\frac{dy}{dx} + 4y = 1$
   (b) $\frac{dy}{dx} + y - \frac{1}{1+e^x} = 0$
5. Solve the differential equations by separation of variables:
   (a) $\frac{dy}{dx} = (1+y^2)x^2$
   (b) $y' = -xy$
6. A tank initially contains 200 gallons of pure water. Then at time $t = 0$ brine containing 5 pounds of salt per gallon of brine is allowed to enter the tank at a rate of 10 gallons per minute and the mixed solution is drained from the tank at the same rate.
   (a) How much salt is in the tank at an arbitrary time t?
   (b) How much salt is in the tank after 30 minutes?
7. Sketch the direction field for $y' + y = 2$ at the gridpoints (x, y), where $x = 0, 1, 2, 3, 4$ and $y = 0, 1, 2, 3, 4$.
8. Suppose that a body moves along an s-axis through a resistive medium in such a way that the velocity $v = v(t)$ decreases at a rate that is twice the square of the velocity.
   (a) Find a differential equation whose solution is the velocity $v(t)$.
   (b) Find a differential equation whose solution is the position $s(t)$.