HOMEWORK 6 - MATH 152

DUE DATE: Monday, November 15
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. One part of each homework problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the Maclaurin polynomials of orders 0,1,2,3 and 4 and then the general Maclaurin polynomial of order n for the functions

   (a) $f(x) = \frac{1}{1+x}$

   (b) $g(x) = xe^x$

2. Show that the n-th Taylor polynomial for $\sinh x$ about $x = \ln 4$ is
$$\sum_{k=0}^{n} \frac{16 - (-1)^k}{8k!}(x - \ln 4)^k$$

3. Write out the first five terms of the following sequences, determine if the sequence converges and, if so, find its limit.

   (a) $\{\ln(\frac{1}{n})\}_{n=1}^{+\infty}$

   (b) $\{n \sin(\frac{\pi}{n})\}_{n=1}^{+\infty}$

   (c) $\{\frac{n}{2^n}\}_{n=1}^{+\infty}$

   (d) $\{\sqrt{n^2+3n} - n\}_{n=1}^{+\infty}$

4. Starting with $n = 1$, and considering the even and odd terms separately, find a formula for the general term of the following sequences. Then determine whether the sequences converge and, if so, find the limits.

   (a) $1, \frac{1}{2^2}, 3, \frac{1}{2^4}, 5, \frac{1}{2^6}, \ldots$

   (b) $1, \frac{1}{3}, \frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{1}{7}, \frac{1}{7}, \ldots$

5. Consider the sequence
$$a_1 = \sqrt{6}, a_2 = \sqrt{6+\sqrt{6}}, a_2 = \sqrt{6+\sqrt{6+\sqrt{6}}}, \ldots$$
   Find a recursion formula for a_{n+1} and, assuming that the sequence converges, find the limit.

6. Consider the sequence $\{a_n\}_{n=1}^{+\infty}$ whose n-th term is $a_n = \frac{1}{n}\sum_{k=1}^{n}\frac{1}{1+(k/n)}$. Show that $\lim_{n\to\infty} a_n = \ln 2$ by interpreting a_n as the Riemann sum of a definite integral.

7. Use the indicated methods to show that the following sequences are monotone:

   (a) $\{\frac{n}{4n-1}\}_{n=1}^{+\infty}$ and $\{n - n^2\}_{n=1}^{+\infty}$ (compute $a_{n+1} - a_n$)

(b) $\{\frac{2^n}{1+2^n}\}_{n=1}^{+\infty}$ and $\{\frac{5^n}{2^{(n^2)}}\}_{n=1}^{+\infty}$ (compute $\frac{a_{n+1}}{a_n}$)

(c) $\{ne^{-2n}\}_{n=1}^{+\infty}$ and $\{\tan^{-1} n\}_{n=1}^{+\infty}$ (use differentiation of the corresponding function)

8. Let $\{a_n$ be the sequence defined recursively by $a_1 = 1$ and $a_{n+1} = \frac{1}{2}(a_n + \frac{3}{a_n})$ for $n \geq 1$.

(a) Show that $a_n \geq \sqrt{3}$ for $n \geq 2$ by finding the minimum value of $\frac{1}{2}(x + \frac{3}{x})$ for $x > 0$.

(b) Show that $\{a_n\}$ is eventually increasing by examining either $a_{n+1} - a_n$ or $\frac{a_{n+1}}{a_n}$ and using part (a).

(c) Show that $\{a_n\}$ converges and find its limit.