HOMEWORK 9 - MATH 112

DUE DATE: Thursday, December 8 INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. One part of each problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Evaluate the improper integrals:

(a)
$$\int_0^\infty \frac{5}{e^{2x}} dx$$
 (b) $\int_{-\infty}^0 \frac{x}{x^2 + 1} dx$

2. Evaluate the improper integrals:

(a)
$$\int_0^1 \frac{1}{x^2} dx$$
 (b) $\int_3^5 \frac{1}{x^2 \sqrt{x^2 - 9}} dx$

3. Use implicit differentiation to verify that the equation is a solution of the differential equation for any value of C.

(a)
$$y^2 + 2xy - x^2 = C$$
, $(x+y)y' - x + y = 0$

(b)
$$x^2 - y^2 = C$$
, $y^3y'' + x^2 - y^2 = 0$

4. Verify that the general solution satisfies the differential equation. Then find the particular solution that satisfies the initial condition.

(a) General Solution: $y = C_1 x + C_2 x^3$

Differential Equation: $x^2y'' - 3xy' + 3y = 0$

Initial Condition: y = 0 and y' = 4 when x = 2.

(b) General Solution: $y = (C_1 + C_2x + \frac{1}{12}x^4)e^{2x}$ Differential Equation: $y'' - 4y' + 4y = x^2e^{2x}$

Initial Condition: y = 2 and y' = 1 when x = 0.

5. Use integration to find the general solution of the differential equation:

(a)
$$\frac{dy}{dx} = \frac{1}{2 - 7x}$$
 (b) $\frac{dy}{dx} = x\sqrt{3x - 5}$ (c) $\frac{dy}{dx} = 11xe^{3x}$

6. Use separation of variables to find the general solution of the differential equation

(a)
$$\frac{dy}{dx} = x^2y$$
 (b) $(1+y)\frac{dy}{dx} - 4x = 0$ (c) $\frac{dy}{dx} = \frac{x^2+2}{3y^2}$ (d) $yy' - 2xe^x = 0$

7. Use the given initial condition to find the particular solution of the differential equation.

(a)
$$yy' - e^x = 0$$
, $y = 4$ when $x = 0$

(b)
$$\sqrt{x} + \sqrt{y}y' = 0$$
, $y = 4$ when $x = 1$

(c)
$$\frac{dy}{dx} = x^2(1+y)$$
, $y = 3$ when $x = 0$.

8. Solve the differential equations

(a)
$$\frac{dy}{dx} + 5y = 15$$
 (b) $\frac{dy}{dx} + 3y = e^{-3x}$ (c) $\frac{dy}{dx} + \frac{2y}{x} = 3x + 1$ (d) $xy' + y = x^2 \ln x$

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