EXAM 4 - MATH 140

DATE: Monday, November 21
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 3 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Show the following trigonometric identities:
   (a) $\frac{1-\tan^2\theta}{1+\tan^2\theta} + 1 = 2\cos^2\theta$
   (b) $\frac{\cos^2\theta-\sin^2\theta}{1-\tan^2\theta} = \cos^2\theta$
2. Find the exact value of the expressions $\sin(\alpha-\beta)$ and $\cos(\alpha-\beta)$ if
$$\tan\alpha = \frac{5}{12},\ \pi < \alpha < \frac{3\pi}{2}, \quad \text{and} \quad \sin\beta = -\frac{1}{2},\ \pi < \beta < \frac{3\pi}{2}.$$
3. Solve the following trigonometric equations, where $0 \leq \theta < 2\pi$.
   (a) $(\cot\theta + 1)(\csc\theta - \frac{1}{2}) = 0$
   (b) $\cos(2\theta) + 5\cos\theta + 3 = 0$
4. George needs to determine the height of a tree before cutting it down to be sure that it will not fall on a nearby fence. The angle of elevation of the tree from one position on a flat path from the tree is 30° and from a second position 40 feet farther along this path it is 20°. What is the height of the tree? (You are given that $\sin(20^\circ) \cong \frac{1}{3}$.)
5. Solve the triangle $\triangle ABC$ if you know that $\alpha = 120^\circ, b = 4$ and $c = 1$.
6. Prove the following formula for the area A of a triangle
$$A = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}.$$