HOMEWORK 1 - MATH 151
DUE DATE: Monday, September 18
INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the domain of the function $f(x) = \frac{1}{\sqrt{7x-x^2}}$.

2. Sketch the graph of the piece-wise defined function $f(x) = \begin{cases} -x^2+1, & \text{if } x < -1 \\ x-5, & \text{if } x \geq -1 \end{cases}$

3. Express the perimeter of a rectangle, with area 25 square meters as a function of the length of one of its sides.

4. Perform the even/odd tests on the function $f(x) = \frac{x^3}{x^6-7}$. Show your work in both cases and state your conclusions clearly.

5. Use your knowledge of the graph of $f(x) = x^2$ and of transformations (graphing techniques) to obtain step-by-step the graph of $g(x) = (x+3)^2 - 4$. Show all transformations used clearly.

6. Let $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+2}$. Find the domain of f, the domain of g, a formula for the function $g \circ f$ and the domain of $g \circ f$. Show all steps clearly.

7. Give at least two different decompositions of the function $h(x) = \sqrt{(x+3)^2+1}$ as a composite of two functions $h = g \circ f$.

8. Sketch the graph of the piece-wise defined function

$$f(x) = \begin{cases} x^3, & \text{if } x \leq -1 \\ -x, & \text{if } -1 < x < 1 \\ x^2 - 2 & \text{if } x > 1 \end{cases}$$

   Find $\lim_{x\to -1^-} f(x), \lim_{x\to -1^+} f(x), \lim_{x\to 1^-} f(x), \lim_{x\to 1^+} f(x)$. Which conclusions can you draw from these limits about $\lim_{x\to -1} f(x)$ and $\lim_{x\to 1} f(x)$?

9. Evaluate the limits $\lim_{x\to 2}(x^2+3x-1), \lim_{x\to 1}\sqrt{x^4+3x+12}$ using carefully the relevant limit properties without omitting any steps.

10. Evaluate the limits $\lim_{x\to -3}\frac{x^2-x+6}{x+3}$, $\lim_{h\to 0}\frac{(7+h)^2-49}{h}$ and $\lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7}$.