

HOMEWORK 4 - MATH 151

DUE DATE: Monday, October 23

INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Use your knowledge of the graph of $f(x) = e^x$ and transformations to sketch the graph of $g(x) = 2(1 - e^{-x})$.
(b) Starting with the graph of $y = (\frac{1}{2})^x$ write the equation of the graph that results from it by applying the following transformations sequentially: shift 2 units downward, reflect about the x -axis, shift 5 units to the right, stretch horizontally by a factor of 3, shift 5 units to the right and reflect about the y -axis.
2. Find the domains of the functions $f(x) = \frac{1}{1-e^x}$ and $g(x) = \sqrt{1-3^x}$.
3. Find the limits $\lim_{x \rightarrow \infty} \frac{e^{7x} - e^{-7x}}{e^{7x} + e^{-7x}}$, $\lim_{x \rightarrow \infty} \frac{2+5^x}{7-5^x}$, $\lim_{x \rightarrow \infty} (e^{-5x} \cos(3x))$.
4. Find formulas for the inverses of the functions $f(x) = \sqrt{2-7x}$, $g(x) = \frac{2x-1}{5x+3}$, $h(x) = \ln(x+7)$ and $k(x) = \frac{1+e^x}{1-e^x}$.
5. Find a formula for $f^{-1}(x)$, determine the domain and range of f^{-1} and sketch the graphs of f and f^{-1} on the same system of coordinate axes:
 - (a) $f(x) = \sqrt{x-3}$
 - (b) $f(x) = 9 - x^2, 0 \leq x \leq 3$
6. Find $(f^{-1})'(a)$ if
 - (a) $f(x) = x^5 - x^3 + 2x$ at $a = 2$
 - (b) $f(x) = \sqrt{x^3 + x^2 + x + 1}$ at $a = 2$.
7. Find the exact value of the expressions $\log_{1/2} 64$, $\log_8 \frac{1}{64}$ and $2^{(\log_2 3 + \log_2 5)}$.
8. Use the properties of logarithms to expand or contract as appropriate:
 - (a) $\ln \sqrt{a(b^2 + c^2)}$
 - (b) $\ln x + a \ln y - b \ln z$
 - (c) $\ln \frac{3x^2}{(x+5)^3}$
9. (a) Solve the equations $e^{2x+3} - 7 = 0$ and $\ln x + \ln(x-1) = 1$ for x .
(b) Let $f(x) = \sqrt{3 - e^{2x}}$ and $g(x) = \ln(2 + \ln x)$. Find the domains of f and g , formulas for $f^{-1}(x)$ and $g^{-1}(x)$ and the domains of f^{-1} and g^{-1} .
10. Calculate the following limits:
 - (a) $\lim_{x \rightarrow \frac{\pi}{2}} \ln(\sin x)$
 - (b) $\lim_{x \rightarrow \infty} [\ln(2+x) - \ln(1+x)]$