PRACTICE EXAM 1 - MATH 151
DATE: Friday, September 22
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} 2x + 5, & \text{if} \quad x < -2 \\ -x^2 + 3, & \text{if} \quad -2 \le x < 1 \\ \sqrt{x}, & \text{if} \quad 1 \le x \end{cases}$$

   (a) Sketch carefully the graph of f.

   (b) Find $\lim_{x \to -2^-} f(x)$, $\lim_{x \to -2^+} f(x)$, $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.

   (c) Use part (b) to draw conclusions about $\lim_{x \to -2} f(x)$ and $\lim_{x \to 1} f(x)$.

2. Calculate the following limits:

   (a) $\lim_{x \to -4} \frac{x^2 + 2x - 8}{x^2 + 7x + 12}$

   (b) $\lim_{x \to 0} \frac{x}{\sqrt{1 + 3x} - 1}$

3. Prove that $\lim_{x \to 0^+} \sqrt{x}[1 + \cos^2\left(\frac{5\pi}{x}\right)] = 0$. Give all details and all relevant explanations.

4. Calculate the following limits:

   (a) $\lim_{x \to 0} \frac{\sin^2 5x}{x^2}$

   (b) $\lim_{x \to 0} \frac{x}{\tan x}$

5. Give an answer with detailed explanations:

   (a) Is the function $f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1}, & \text{if} \quad x \ne 1 \\ 1, & \text{if} \quad x = 1 \end{cases}$ continuous at $x = 1$?

   (b) Is the function $g(x) = \begin{cases} \sin x, & \text{if} \quad x < \frac{\pi}{4} \\ \cos x, & \text{if} \quad x \ge \frac{\pi}{4} \end{cases}$ continuous at $x = \frac{\pi}{4}$?

6. Find the numbers at which $f(x) = \begin{cases} x + 2, & \text{if} \quad x < 0 \\ 2x^2, & \text{if} \quad 0 \le x \le 1 \\ 2 - x, & \text{if} \quad x > 1 \end{cases}$ is discontinuous. At which of these points is f continuous from the right, from the left or neither? It is up to you to use either the geometric or the algebraic technique but you should provide detailed explanations either way.