PRACTICE EXAM 2 - MATH 151

DATE: Friday, October 13

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Use the **limit definition** of the derivative to compute the derivatives of the following functions at the designated points:
   (a) $f(x) = \frac{1}{x-5}$ at $x = 3$.
   (b) $g(x) = \sqrt{x-7}$ at $x = 11$.

2. A particle moves according to the law of motion $s(t) = t^3 - 9t^2 + 15t + 10$, $t \geq 0$.
   (a) Find the velocity of the particle at time $t = a$. (1 point)
   (b) Find the acceleration of the particle at time $t = a$. (1 point)
   (c) When is the particle at rest? (1 point)
   (d) When is the particle moving in the positive direction? (2 points)

3. Compute the following derivatives:
   (a) $f(x) = (3x+7)^4 \cdot \sqrt{5x^2-3}$ (2 points)
   (b) $g(x) = \frac{\cos x}{x^3-2x+5}$ (2 points)
   (c) Find the equation of the tangent line to the graph of $f(x) = \frac{1}{1+x^2}$ at the point $(-1, \frac{1}{2})$. (1 point)

4. Compute the following derivatives:
   (a) $f(x) = \sqrt{1+\tan^2 x}$ (2 points)
   (b) $g(x) = x^3 \cos^2(5x)$ (2 points)
   (c) $h(x) = (\frac{x^2}{x-1})^7$ (1 point)

5. (a) Find $\frac{dy}{dx}$ if $y^5 + x^2y^3 = 1 + x^4y$. (2 points)
   (b) Find an equation for the tangent line to the graph of $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$. (3 points)

6. A kite 100 feet above the ground moves horizontally at a speed of 8 feet per second. At what rate is the angle between the string and the horizontal decreasing when 200 feet of string has been let out?
   (a) Denote by θ the angle between the string and the horizontal, by x the horizontal distance from the kite to the vertical at the beginning of the string. Write an equation relating θ and x. (2 points)
   (b) Implicitly differentiate both sides of the equation with respect to time and solve for $\frac{d\theta}{dt}$. (2 points)
   (c) Answer the question posed by the problem. (1 point)