PRACTICE EXAM 3 - MATH 151
DATE: Friday, November 3
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Find $(f^{-1})'(2)$ if $f(x) = x^5 - x^3 + 2x$. (2 points)

   (b) Find $f^{-1}(x)$ if $f(x) = \ln(7 + \ln x)$. (2 points) Find also the domains and the ranges of both f and f^{-1}. (1 point)

2. (a) Find an equation for the tangent line to the graph of $f(x) = \ln \ln x$ at $x = e$. (2 points)

   (b) Differentiate the following functions:

   $$(a)\ f(x) = xe^{-x^2} \quad (1.5 \text{ points}) \quad (b)\ f(x) = \ln(x^4 \sin^2 x) \quad (1.5 \text{ points})$$

3. (a) Compute the derivative $f'(x)$ of $f(x) = \sqrt{x}^x$. (2 points)

   (b) Find an equation of the tangent line to the curve $xe^y + ye^x = 1$ at the point $(0, 1)$. (3 points)

4. A thermometer is taken from a room where the temperature is 20 degrees Celsius to the outdoors, where the temperature is 5 degrees. After 1 minute the thermometer reads 12 degrees. Find the reading of the thermometer after t minutes. (2 points) Find what the reading will be after 1 additional minute. (1 point) When will the thermometer read 6 degrees? (2 points)

5. Find the derivatives of the following functions:

   (a) $f(x) = \cos^{-1}(e^{2x})$ (1 point)

   (b) $g(x) = \tan^{-1}(x - \sqrt{1 + x^2})$ (2 points)

   (c) $h(x) = \sinh(\cosh x)$ (2 points)

6. Calculate the following limits:

   (a) $\lim_{x \to \infty} \frac{\ln \ln x}{x}$ (1 point)

   (b) $\lim_{x \to -\infty} x^2 e^x$ (2 points)

   (c) $\lim_{x \to 0^+} x^{x^2}$ (2 points)