

# EXAM 1 - MATH 112

DATE: Tuesday, September 25

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Let  $f(x) = \frac{1}{5-x}$  and  $g(x) = \sqrt{x-2}$ . Find a formula for  $(g \circ f)(x)$  and simplify. (2 points)  
Find  $\text{Dom}(f)$  and  $\text{Dom}(g)$ . (1 point) Find the domain of  $g \circ f$ . (2 points)
2. Find  $f^{-1}(x)$  if  $f(x) = \sqrt[3]{\frac{x-2}{2x-5}}$ . (3 points) To verify your answer check that  $(f \circ f^{-1})(x) = x$ . (2 points)
3. Find the following limits:
  - (a)  $\lim_{x \rightarrow -1} \frac{7x+8-x^2}{3x^2+7x+4}$  (2.5 point)
  - (b)  $\lim_{x \rightarrow 5} \frac{x-5}{\sqrt{x-4}-1}$  (2.5 points)
4. Consider the function  $f(x) = \begin{cases} 4, & \text{if } x \leq -1 \\ -x^2 + 5, & \text{if } -1 < x \leq 2 \\ -2x + 8, & \text{if } x > 2 \end{cases}$ 
  - (a) Roughly sketch the graph of  $y = f(x)$ . (2 points)
  - (b) Find the  $\lim_{x \rightarrow -1} f(x)$  and  $\lim_{x \rightarrow 2} f(x)$ , if they exist. (2 points)
  - (c) Is  $f$  continuous at  $x = -1$ ? How about at  $x = 2$ ? Please, explain. (1 point)
5. Use the **limit definition of the derivative** to compute the equation of the tangent line to the graph of  $f(x) = x^3 - 2$  at the point  $x = 2$ . (5 points)
6. Find all points on the graph of  $f(x) = \frac{8}{3}x^3 - 28x + 5$  at which the tangent lines to the graph have slope 4. (5 points)