HOMEWORK 4 - MATH 112
DUE DATE: Monday, October 15
INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the eight problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. A spherical balloon is inflated with gas at a rate of 20 cubic feet per minute. How fast is the radius of the balloon changing at the instant when the radius is 2 feet?

2. A 25-foot ladder is leaning against a house. The base of the ladder is pulled away from the house at the rate of 2 feet per second. How fast is the top of the ladder sliding down the wall when the base is 7 feet from the house?

3. Find the domain, the critical numbers and the open intervals on which the given function is increasing or decreasing.

   (a) $f(x) = x^3 - 6x^2$
   (b) $g(x) = \sqrt{4 - x^2}$
   (c) $f(x) = \frac{2x}{16 - x^2}$

4. Create the table for the first derivative in order to find the monotonicity and the relative extrema of the following functions.

   (a) $f(x) = -4x^2 + 4x + 1$
   (b) $g(x) = \frac{1}{5}x^5 - x$
   (c) $h(x) = x^4 - 32x + 4$

5. Find the absolute extrema of the function in the given closed interval:

   (a) $f(x) = x^3 - 12x$ on $[0, 4]$
   (b) $g(x) = \frac{x}{x-2}$ on $[3, 5]$

6. The quantity demanded x for a product is inversely proportional to the cube of the price p for $p > 1$. When the price is \$10 per unit, the quantity demanded is 8 units. The initial (fixed) cost is \$100 and the cost per unit is \$4. What price will yield a maximum profit?

7. Use the second derivative test to find all relative extrema of the function. Write "Not Applicable" when the test is inconclusive.

   (a) $f(x) = x^4 - 4x^3 + 2$
   (b) $g(x) = x + \frac{4}{x}$
   (c) $h(x) = \frac{x}{x^2 - 1}$

8. Make a sign table for both the first and the second derivative to find both the intervals of monotonicity and the intervals of concavity of the following functions.

   (a) $f(x) = (x - 1)^3(x - 5)$
   (b) $g(x) = (1 - x)(x - 4)(x^2 - 4)$