HOMEWORK 6 - MATH 112
DUE DATE: Monday, November 5
INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the eight problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Solve the following exponential equations:

   (a) $(\frac{1}{3})^{3x+1} = 27$

   (b) $4^2 = (x+2)^2$

   (c) $5^{x^2} = 25^{3x} \cdot 5^7$

2. After t years, the value of a car that originally cost \$16,000 depreciates so that each year it is worth $3/4$ of its value for the previous year. Find a model for $V(t)$, the value of the car after t years. What is the value of the car 6 years after it was purchased?

3. How much should be deposited now in an account paying interest rate 4% compounded every two months so as to have $10,000$ in the account in 5 years time?

4. Find the derivatives of the following functions

   (a) $f(x) = e^{1-x^2}$

   (b) $g(x) = (4x^2 - 7x^2 + 1)e^{7x}$

   (c) $h(x) = \frac{(e^x + e^{-x})^4}{2}$

5. Determine the equation of the tangent line to the graph of

   (a) $f(x) = xe^{-2x}$ at $(1, \frac{1}{e^2})$

   (b) $e^{xy} + x^2 - y^2 = 10$ at $(3, 0)$.

6. Find the domain, the intercepts, the relative extrema, the intervals of monotonicity, the inflection points and the intervals of concavity and then sketch the graph of the exponential function $f(x) = x^2 e^{-x}$.

7. Find the derivative of the following functions:

   (a) $f(x) = \ln \sqrt{x^4 - 4x}$

   (b) $f(x) = \ln \frac{x^2}{x^2+1}$

   (c) $f(x) = \frac{\ln x}{x^2}$

   (d) $x^2 - 3\ln y + y^2 = 10$

   (e) $4xy + \ln(x^2 y) = 7$

8. Analyze the function $f(x) = \frac{\ln x}{x}$. (Domain, intercepts, asymptotes, monotonicity, extrema, concavity, inflection points, graph)