

EXAM 1 - MATH 151

DATE: Friday, September 21

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Starting from the graph of $f(x) = x^3$ give the step-by-step transformations that need to be followed to graph the function $g(x) = \frac{1}{2}(2-x)^3 + 1$. (You do not need to do the actual graphing.) (3 points)
(b) Graph the function $g(x)$ of the previous part. (2 points)
2. Suppose that $f(x) = \frac{4}{x+2}$ and $g(x) = \sqrt{x+2}$.
 - (a) Find a formula for $(g \circ f)(x)$ and simplify. (2 points)
 - (b) Find the domains $\text{Dom}(f)$ and $\text{Dom}(g)$. (1 point)
 - (c) Find the domain of $g \circ f$. (2 points)

3. Graph the piece-wise defined function $f(x) = \begin{cases} (x+1)^2 + 1, & \text{if } x \leq -1 \\ -x^2, & \text{if } -1 \leq x \leq 2 \\ 2x - 8, & \text{if } x > 2 \end{cases}$ (3 points)

Then, find the following limits

- (a) $\lim_{x \rightarrow -1^-} f(x), \lim_{x \rightarrow -1^+} f(x)$ (1 point)
 - (b) $\lim_{x \rightarrow 2^-} f(x), \lim_{x \rightarrow 2^+} f(x)$ (1 point)
4. Find the following limits:
 - (a) $\lim_{x \rightarrow -1} \frac{x^2 - 4x}{x^2 - 3x - 4}$ (1 point)
 - (b) $\lim_{x \rightarrow 7} \frac{\sqrt{x+2}-3}{x-7}$ (2 points)
 - (c) $\lim_{x \rightarrow -6^-} \frac{2x+12}{|x+6|}$ (2 points)
 5. Evaluate the following limits (**giving full explanations** for your answers):
 - (a) $\lim_{x \rightarrow 1} [(x^4 - 1) \cos(\frac{2}{x-1})]$ (3 points)
 - (b) $\lim_{x \rightarrow 1} f(x)$ if $2x + 3 \leq f(x) \leq x^2 + 4$. (2 points)
 6. Find the following trigonometric limits:
 - (a) $\lim_{x \rightarrow 0} \frac{\sin^2 5x}{x^2}$ (2 points)
 - (b) $\lim_{x \rightarrow 0} \frac{x}{\tan x}$ (3 points)