

EXAM 2 - MATH 151

DATE: Friday, October 12

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Consider the function

$$f(x) = \begin{cases} \frac{x^4 - x^3 + x - 1}{x - 1}, & \text{if } x < 1 \\ 4, & \text{if } x = 1 \\ -x^2 + 3, & \text{if } x > 1 \end{cases}$$

Find $\lim_{x \rightarrow 1} f(x)$. (3 points) Is f continuous at $x = 1$? Explain. If not, what kind of discontinuity does f have at $x = 1$? Explain. (2 points)

2. Consider the function $f(x) = \sqrt{x - 5} - \frac{1}{x + 3}$.

(a) Find the domain of f . (2 points)

(b) Prove (in detail) that the equation $f(x) = 0$ has at least one real root. (3 points)

3. Compute the following limits:

(a) $\lim_{x \rightarrow -5^+} \frac{2x^2 - 3x - 2}{x^2 + 6x + 5}$ (2 points)

(b) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 2x} - \sqrt{x^2 - 7x})$ (3 points)

4. (a) Provide both forms of the limit definition of the derivative of a function $y = f(x)$ at a point $x = a$. (2 points)

(b) Use either of the two forms of the limit definition to find the derivative of $f(x) = \frac{2x-3}{5x-7}$ at $x = 2$. (3 points)

5. Find an equation of the tangent line to the curve $y = (1 + x) \cos x$ at $x = 0$. (2 points)

6. Find $f''(1)$ if $f(x) = \frac{x^2}{1+x}$. (3 points)