

HOMEWORK 1 - MATH 151

DUE DATE: Monday, September 17

INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the domain of the function $f(x) = \frac{1}{\sqrt{7x-x^2}}$.
2. Sketch the graph of the piece-wise defined function $f(x) = \begin{cases} -x^2 + 1, & \text{if } x < -1 \\ x - 5, & \text{if } x \geq -1 \end{cases}$
3. Express the perimeter of a rectangle, with area 25 square meters as a function of the length of one of its sides.
4. Perform the even/odd tests on the function $f(x) = \frac{x^3}{x^6-7}$. Show your work in both cases and state your conclusions clearly.
5. Use your knowledge of the graph of $f(x) = x^2$ and of transformations (graphing techniques) to obtain step-by-step the graph of $g(x) = (x+3)^2 - 4$. Show all transformations used clearly.
6. Let $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+2}$. Find the domain of f , the domain of g , a formula for the function $g \circ f$ and the domain of $g \circ f$. Show all steps clearly.
7. Give at least two different decompositions of the function $h(x) = \sqrt{(x+3)^2 + 1}$ as a composite of two functions $h = g \circ f$.
8. Sketch the graph of the piece-wise defined function

$$f(x) = \begin{cases} x^3, & \text{if } x \leq -1 \\ -x, & \text{if } -1 < x < 1 \\ x^2 - 2 & \text{if } x > 1 \end{cases}$$

Find $\lim_{x \rightarrow -1^-} f(x)$, $\lim_{x \rightarrow -1^+} f(x)$, $\lim_{x \rightarrow 1^-} f(x)$, $\lim_{x \rightarrow 1^+} f(x)$. Which conclusions can you draw from these limits about $\lim_{x \rightarrow -1} f(x)$ and $\lim_{x \rightarrow 1} f(x)$?

9. Evaluate the limits $\lim_{x \rightarrow 2} (x^2 + 3x - 1)$, $\lim_{x \rightarrow 1} \sqrt{x^4 + 3x + 12}$ using carefully the relevant limit properties without omitting any steps.
10. Evaluate the limits $\lim_{x \rightarrow -3} \frac{x^2 - x + 6}{x + 3}$, $\lim_{h \rightarrow 0} \frac{(7+h)^2 - 49}{h}$ and $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7}$.