HOMEWORK 4 - MATH 151
DUE DATE: Monday, October 22
INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Differentiate the following functions:

$$(a)\ f(x) = x^3 \sin x \quad (b)\ f(x) = \left(\frac{1}{x^2} - \frac{3}{x^4}\right)(x + 5x^3) \quad (c)\ f(x) = \frac{x^2}{3x^2 - 2x + 1}$$

$$(d)\ f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \quad (e)\ f(x) = \frac{1+\cos x}{x + \sin x}$$

2. Find the derivative of the following functions:

$$(a)\ f(x) = \sqrt{\frac{x-1}{x+1}} \quad (b)\ f(x) = \sin\sqrt{1+x^2} \quad (c)\ f(x) = (1+\cos^2 x)^9 \quad (d)\ f(x) = \cos(\cos(\cos x))$$

3. Find all points on the graph of $f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

4. Find the derivative dy/dx:
   (a) $x^2 - 2xy + y^3 = c$
   (b) $y^5 + x^2y^3 = 1 + x^4y$
   (c) $y \sin(x^2) = x \sin(y^2)$

5. (a) Find an equation of the tangent line to the curve of $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.
   (b) Find all points on the curve $x^2y^2 + xy = 2$ where the slope of the tangent line is -1.

6. Each side of a square is increasing at a rate of 6 cm/sec. At what rate is the area of the square increasing when the area of the square is 16 square centimeters?

7. A plane flying horizontally at an altitude of 1 mile and a speed of 500 mi/hr passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.

8. A paper cup has the shape of a cone with height 10 cm and radius 3 cm at the top. If water is poured into the cup at the rate of 2 cubic centimeters per second, how fast is the water level rising when the water is 5 cm deep?

9. Use a linear approximation to estimate the numbers $\sqrt{99.8}$ and $\frac{1}{1002}$.

10. Find the differentials of the functions
    (a) $y = x^2 \sin 2x$
    (b) $y = \frac{1}{x+1}$.