HOMEWORK 6 - MATH 151
DUE DATE: Monday, November 5
INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Compute the following derivatives:

$$(a)\ f(x) = \log_5(xe^x) \quad (b)\ f(x) = \frac{1+\ln x}{1-\ln x} \quad (c)\ f(x) = \ln(x^7 \sin^3 x)$$

$$(d)\ f(x) = \sqrt[3]{x}\, e^{9x} \quad (e)\ f(x) = \frac{1-xe^x}{x+e^x}$$

2. Use logarithmic differentiation to compute the derivative of

$$(a)\ y = \sqrt{x}e^{x^2}(x^2+1)^{12} \quad (b)\ y = \sqrt[5]{\frac{x^3+1}{x^3-1}} \quad (c)\ y = (\sin x)^{\ln x}$$

3. (a) Find y' if $e^{x^2 y} = x + y$
   (b) Find an equation of the tangent line to the curve $xe^y + ye^x = 1$ at $(0,1)$.
4. A sample of tritium-3 decayed to 94.5% of its original amount after a year. What is its half-time? How long would it take the sample to decay to 20% of its original amount?
5. A curve passes through the point $(0,5)$ and has the property that the slope of the curve at every point P is twice the y-coordinate of P. What is the equation of the curve?
6. (a) Prove that $(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$.
   (b) If $g(x) = x\sin^{-1}(x/4) + \sqrt{16-x^2}$, find $g'(2)$.
7. Find the derivatives of the following functions

$$(a)\ f(x) = \sqrt{1-x^2}\arcsin x \quad (b)\ f(x) = x\ln(\arctan x) \quad (c)\ f(x) = x\cos^{-1} x - \sqrt{1-x^2}$$

8. Prove the identities

$$(a)\ \cosh x - \sinh x = e^{-x} \quad (b)\ \cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$

$$(c)\ \sinh 2x = 2\sinh x \cosh y$$

9. Use the definitions of the hyperbolic functions to find each of the following limits:

$$(a)\ \lim_{x\to-\infty} \tanh x \quad (b)\ \lim_{x\to-\infty} \sinh x \quad (c)\ \lim_{x\to 0^-} \coth x$$

10. Find the derivatives

$$(a)\ f(x) = \sinh x \tanh x \quad (b)\ f(x) = \ln(\sinh x) \quad (c)\ f(x) = \sinh(\cosh x)$$