

PRACTICE EXAM 2 - MATH 151

DATE: Friday, October 12

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the values of the parameters a and b for which the function

$$f(x) = \begin{cases} ax^2 - b, & \text{if } x < 2 \\ ax, & \text{if } x = 2 \\ 7 - bx, & \text{if } x > 2 \end{cases}$$

is continuous at $x = 2$. (3 points) Then make a rough sketch of its graph **showing** all crucial points.

2. (a) Prove that the equation $x^5 - x^2 + 2x + 3 = 0$ has at least one real root. (3 points)
(b) Use continuity to evaluate the limit $\lim_{x \rightarrow -\pi} \cos(x + \sin x)$. (2 points)
3. Find the following limits carefully:
- (a) $\lim_{x \rightarrow -2} \frac{x^2 + x - 12}{x^2 + x - 2}$ (2 points)
(b) $\lim_{x \rightarrow +\infty} (\sqrt{9x^2 + x} - 3x)$ (2 points)
(c) $\lim_{x \rightarrow -\infty} \frac{3x+5}{x-4}$ (1 point)
4. Use the limit definition of the derivative to compute $f'(a)$ if $f(x) = \frac{5}{x-1}$. (5 points)
5. Find the domain of $f(x) = x^2 - \sqrt{x}$. (1 point) Compute $f'(x)$. (2 points) Find all points on the graph of $y = f(x)$, where the tangent line to $y = f(x)$ is horizontal. (2 points)
6. Compute the derivatives of the following functions:
- (a) $f(x) = \frac{\sin x}{x^2}$ (2 points)
(b) $f(x) = \cot x$ (2 points)
(c) $f(x) = \sqrt[3]{x^5} \sin x$ (1 point)