PRACTICE EXAM 3 - MATH 151

DATE: Friday, November 2

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Compute the following derivatives:
   (a) $f(x) = \sqrt{1 + \tan^2 x}$ (2 points)
   (b) $g(x) = x^3 \cos^2(5x)$ (2 points)
   (c) $h(x) = (\frac{x^2}{x-1})^7$ (1 point)

2. (a) Find $\frac{dy}{dx}$ if $y^5 + x^2y^3 = 1 + x^4y$. (2 points)
   (b) Find an equation for the tangent line to the graph of $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$. (3 points)

3. A 13-ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/sec, how fast will the foot be moving away from the wall when the top is 5 feet above the ground?
   (a) Denote by h the distance of the top of the ladder from the ground and by x the distance of the foot of the ladder from the wall. Write an equation relating h and x. (2 points)
   (b) Implicitly differentiate both sides of the equation with respect to time and solve for $\frac{dx}{dt}$. (2 points)
   (c) Answer the question posed by the problem. (1 point)

4. A kite 100 feet above the ground moves horizontally at a speed of 8 feet per second. At what rate is the angle between the string and the horizontal decreasing when 200 feet of string has been let out?
   (a) Denote by θ the angle between the string and the horizontal, by x the horizontal distance from the kite to the vertical at the beginning of the string. Write an equation relating θ and x. (2 points)
   (b) Implicitly differentiate both sides of the equation with respect to time and solve for $\frac{d\theta}{dt}$. (2 points)
   (c) Answer the question posed by the problem. (1 point)

5. (a) Find $(f^{-1})'(4)$ if $f(x) = 3 + x + e^x$. (2 points)
   (b) At which point (x, y) on the curve $y = [\ln(x + 4)]^2$ is the tangent horizontal? (3 points)

6. Compute $\frac{dy}{dx}$.
   (a) $y = (\cos x)^x$ (3 points)
   (b) $xe^y = y - 1$ (2 points)