HOMEWORK 3: SOLUTIONS - MATH 111
INSTRUCTOR: George Voutsadakis

Problem 1 *The Revenue R in terms of the number of items produced is given by $R(x) = 10x$ and the cost C by $C(x) = 5x+65$. Find the break-even point and the break-even revenue.*

Solution:

At the break-even point we have $R(x) = C(x)$. Hence $10x = 5x + 65$, which yields $5x = 65$, and therefore $x = 13$. The break-even revenue is thus $R(13) = 10 \cdot 13 = 130$. ■

Problem 2 *The supply S and the demand D in terms of the number of items q are given by $S(q) = \frac{1}{3}q + 4$ and $D(q) = -q + 24$, respectively. Find the equilibrium demand and the equilibrium price.*

Solution:

At the equilibrium point $S(q) = D(q)$. Thus, $\frac{1}{3}q + 4 = -q + 24$, whence $\frac{4}{3}q = 20$, which yields $q = 15$. The equilibrium price is $D(15) = S(15) = -15 + 24 = 9$. ■

Problem 3 *Find the number of solutions of* $3x^2 - 6x + 2 = 0$.

Solution:

For the number of solutions of a quadratic one only has to compute the discriminant $D = b^2 - 4ac$ and check its sign. If $D > 0$, then the quadratic has two different solutions. If $D = 0$, then the quadratic has one double root and if $D < 0$, then the quadratic does not have any real roots. In the present case we have $D = b^2 - 4ac = (-6)^2 - 4 \cdot 3 \cdot 2 = 36 - 24 = 12 > 0$. Hence the quadratic has two different real roots. ■

Problem 4 *Use the quadratic formula to solve* $10x^2 + x - 2 = 0$.

Solution:

Compute the discriminant $D = b^2 - 4ac = 1^2 - 4 \cdot 10 \cdot (-2) = 1 + 80 = 81$. Hence $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{81}}{2 \cdot 10} = \frac{-1 \pm 9}{20}$, whence $x_1 = \frac{2}{5}$ and $x_2 = -\frac{1}{2}$. ■

Problem 5 *Solve the inequality* $x^2 - 8x + 15 \geq 3$.

Solution:

First subtract 3 from both sides to obtain $x^2 - 8x + 12 \geq 0$. The left hand side now factors as $(x - 2)(x - 8) \geq 0$. Therefore, by building the sign table one discovers that $x \leq 2$ or $x \geq 6$ give the solutions for this inequality. ■

Problem 6 *Solve the inequality* $\frac{x+5}{x-7} \leq 0$.

Solution:

This inequality is also solved by constructing the sign table for the fraction $\frac{x+5}{x-7}$. One then sees that the fraction becomes ≤ 0 when $-5 \leq x < 7$. ■

Problem 7 *Find the domain of* $f(x) = |2x - 7|$.

Solution:
Since no denominators or square roots appear in the expression defining $f(x)$ the domain of f is the set $D(f) = \mathbb{R} = (-\infty, +\infty)$ of all real numbers. ■

Problem 8 *Find the domain of* $g(x) = \sqrt{\frac{x^2-4x+4}{x^2+2x-3}}$.

Solution:
The two restrictions that x should obey are, first, that $x^2 + 2x - 3 \neq 0$ and, second, that $\frac{x^2-4x+4}{x^2+2x-3} \geq 0$. To find the x's that obey both we may set up the sign table for the fraction $\frac{x^2-4x+4}{x^2+2x-3} = \frac{(x-2)^2}{(x+3)(x-1)}$. The sign table, if built correctly, would give us $x < -3$ or $x > 1$ as the range of values that satisfy both restrictions simultaneously. Hence $D(g) = \{x : x < -3 \text{ or } x > 1\}$. ■