HOMEWORK 5: SOLUTIONS - MATH 111
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Problem 1 *Create the sign table and graph the function* $f(x) = x^4 - 7x^2 + 12$.

Solution:

We have $f(x) = 0$ implies $(x^2 - 3)(x^2 - 4) = 0$, whence $(x - \sqrt{3})(x + \sqrt{3})(x - 2)(x + 2) = 0$, and therefore the sign table must have four points $-2, -\sqrt{3}, \sqrt{3}$ and 2 and five rows corresponding to $x - \sqrt{3}, x + \sqrt{3}, x - 2, x + 2$ and $f(x)$. The sign of $f(x)$ turns out to be $+$ if $x < -2$, $-$ if $-2 < x < -\sqrt{3}$, $+$ if $-\sqrt{3} < x < \sqrt{3}$, $-$ if $-\sqrt{3} < x < 2$ and, finally $+$ if $x > 2$.

The rough sketch follows:

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Problem 2 *Study the function* $f(x) = \frac{3x-6}{6x-1}$. *(**Studying** here means what we did in class for rational functions: Find the domain, find the x- and y-intercepts, find the horizontal and vertical asymptotes and then roughly plot the graph.)*

Solution:

The domain is $D(f) = \mathbb{R} - \{\frac{1}{6}\}$, since $\frac{1}{6}$ is a root of the denominator. For the x-intercept, set $y = 0$. Then $3x - 6 = 0$, whence $x = 2$. Thus $(2, 0)$ is the x-intercept. For the y-intercept, set $x = 0$. We get $y = 6$, i.e., $(0, 6)$ is the y-intercept. The horizontal asymptote is $y = \frac{3}{6} = \frac{1}{2}$ and the vertical asymptote is $x = \frac{1}{6}$.

The rough sketch follows:

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Problem 3 *Find the equations of the vertical and horizontal asymptotes of the function* $f(x) = \frac{x^2-2x-3}{x^2-7x+10}$.

Solution:

The horizontal asymptote is $y = \frac{1}{1} = 1$. To find the vertical asymptotes, we factor both numerator and denominator into linear factors: $f(x) = \frac{(x-3)(x+1)}{(x-5)(x-2)}$. Thus, the two vertical asymptotes occur at the roots of the denominator: $x = 2$ and $x = 5$. ■

Problem 4 *Graph on the same axes the functions* $f(x) = 7^x, g(x) = 7^{-x}$ *and* $h(x) = -7^x$. *Before graphing compute their values at* $x = -1, x = 0$ *and* $x = 1$ *and depict those* clearly *both on a small table* and *on your graphs.*

Solution:

We have

x	$f(x)$	$g(x)$	$h(x)$
-1	$\frac{1}{7}$	7	$-\frac{1}{7}$
0	1	1	-1
1	7	$\frac{1}{7}$	-7

The graphs follow:

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Problem 5 *Solve the equation* $7^{x^2} = 49^{4x-\frac{7}{2}}$.

Solution:

We have $7^{x^2} = 49^{4x-\frac{7}{2}}$ implies $7^{x^2} = (7^2)^{4x-\frac{7}{2}}$, whence $x^2 = 2(4x - \frac{7}{2})$. Therefore $x^2 = 8x - 7$, which yields $x^2 - 8x + 7 = 0$. This gives $(x - 7)(x - 1) = 0$, i.e., $x = 1$ or $x = 7$. ■

Problem 6 *Solve the equation* $11^{-2x+5} = (\frac{1}{11})^{-2x+3}$.

Solution:

We have $11^{-2x+5} = (\frac{1}{11})^{-2x+3}$ implies $11^{-2x+5} = (11^{-1})^{-2x+3}$, whence $-2x + 5 = -(-2x+3)$. Therefore $-2x+5 = 2x-3$. Hence $4x = 8$, which, finally, yields $x = 2$. ■

Problem 7 *Culture studies in the lab have determined that the population of an organism A as a function of time t is given by $f(t) = e^{t^2}$. At the same time, the population of another organism B in the same culture has been increasing according to the function $g(t) = \sqrt{e}^{16t+40}$. At what time will the two organisms have the same populations in the culture?*

Solution:

We have $e^{t^2} = \sqrt{e}^{16t+40}$, whence $e^{t^2} = (e^{\frac{1}{2}})^{16t+40}$, i.e., $t^2 = \frac{1}{2}(16t+40)$. This gives $t^2 = 8t+20$, whence $t^2 - 8t - 20 = 0$. We thus get $(t-10)(t+2) = 0$, i.e., $t = -2$ or $t = 10$. Since t represents time $t = 10$. ■

Problem 8 *Compute* $\ln(\sqrt[7]{e})$ *and* $\ln(e^{13})$ *without using a calculator.*

Solution:

We get $\ln(\sqrt[7]{e}) = \ln e^{\frac{1}{7}} = \frac{1}{7}$. Similarly, $\ln(e^{13}) = 13$. ■