HOMEWORK 9: SOLUTIONS - MATH 111
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Problem 1 *Consider the experiment of tossing a coin twice and then rolling a die.*

(a) Write the sample space for this experiment.

(b) Give the event "an even number of heads occurs and the die comes up odd". (This is one event!!)

Solution:

$$S = \{(HH1),(HH2),\ldots,(HH6),(HT1),(HT2),\ldots,(HT6),(TH1),(TH2),\ldots,(TH6),(TT1),(TT2),\ldots,(TT6)\}.$$

The given event then is $E = \{(TT1),(TT3),(TT5),(HH1),\ (HH3),(HH5)\}$. ■

Problem 2 *Consider the experiment of drawing a card from an ordinary deck of 52 cards. Find the probability of drawing a black face card.*

Solution:
Let $E =$ "Black Face Card". Then $P(E) = \frac{|E|}{|S|} = \frac{6}{52} = \frac{3}{26}$. ■

Problem 3 *Consider the experiment of successively drawing two balls out of an urn that contains 5 white, 6 red, 12 green and 7 black balls* without replacement. *What is the probability of the first ball being red and the second black? (This is one probability!!)*

Solution:
$P(1stR \cap 2ndB) = P(2ndB|1stR)P(1stR) = \frac{7}{29}\frac{6}{30}$. ■

Problem 4 *Consider a very small community college with 500 students. This fall semester 150 students are taking Business 101 and 120 are taking Math 111. 70 of these students are taking both Business 101 and Math 111. If one of the students is picked at random from the college population, what is the probability that he will be taking Business 101 or Math 111?*

Solution:
$P(B \cup M) = P(B) + P(M) - P(B \cap M) = \frac{150}{500} + \frac{120}{500} - \frac{70}{500} = \frac{200}{500} = 0.4$ ■

Problem 5 *Suppose that for two events E and F in a sample space S you know that $P(E) = 0.4, P(F^c) = 0.3$ and $P((E \cap F)^c) = 0.85$. Can you find $P((E \cup F)^c)$?*

Solution:
We have $P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.4 + 0.7 - 0.15 = 0.95$. Hence $P((E \cup F)^c) = 1 - P(E \cup F) = 1 - 0.95 = 0.05$. ■

Problem 6 *Consider the experiment of rolling two fair dice. What is the probability of the sum being even given that at least one of the dice showed an odd face?*

Solution:
$P(\text{sum even}|\text{at least 1 odd}) = \frac{P(\text{sum even and at least 1 odd})}{P(\text{at least 1 odd})} = \frac{9}{27} = \frac{1}{3}$. ■

Problem 7 *A teacher has found that the probability that a student studies for a test is 0.6, the probability that a student gets a good grade on a test is 0.7 and the probability that both occur is 0.52. Are these events independent?*

Solution:
Let $E =$ "study" and $F =$ "good grade". Then $P(E)P(F) = 0.6 \cdot 0.7 = 0.42 \neq 0.52 = P(E \cap F)$. Thus E and F are not independent events. ■

Problem 8 *Consider the experiment of drawing successively two cards out of an ordinary deck of 52 cards without replacement. What is the probability of the second card being a red King given that the first card was a black ace?*

Solution:
$P(2ndRedK|1stBlackA) = \frac{2}{51}$. ■