

## EXAM 1 - MATH 325

Thursday, February 6, 2003

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 10 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) State the Law of Sines.  
(b) Let  $AD$  be the angle bisector of a triangle  $ABC$ . Use the law of sines to prove that  $\frac{AB}{AC} = \frac{BD}{DC}$ .
2. (a) State Ceva's Theorem.  
(b) Show that the altitudes of an acute triangle are concurrent.
3. (a) Consider the incircle of the triangle  $ABC$  touching  $BC$ ,  $AC$  and  $AB$  at the points  $X$ ,  $Y$  and  $Z$ , respectively. Denote by  $x$  the length of  $AY$  and by  $s$  the semiperimeter of  $ABC$ . Prove that  $x = s - a$ .  
(b) Show that, if  $r$  is the inradius of  $ABC$ , then the area  $(ABC) = sr$ .
4. (a) Define the orthic triangle of a triangle  $ABC$ .  
(b) Show that the altitudes of an acute-angled triangle are the angle bisectors of its orthic triangle.
5. (a) Give the definitions of orthocenter and circumcenter of a triangle.  
(b) Given a triangle  $ABC$ , draw line  $WV$  through  $A$  parallel to  $BC$ , line  $UW$  through  $B$  parallel to  $AC$  and line  $UV$  through  $C$  parallel to  $AB$ . Show that the orthocenter of  $ABC$  is the circumcenter of  $UVW$ .