

## EXAM 2 - MATH 325

Thursday, February 27, 2003

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Read each problem very carefully before starting to solve it. Each question is worth 10 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) List the nine points that give the nine-point circle its name.  
(b) Let  $H$  be the orthocenter and  $O$  the circumcenter of a triangle  $ABC$ . Show that  $\widehat{HAO} = |\hat{B} - \hat{C}|$ .
2. (a) Give the definition of a pedal triangle.  
(b) Let  $ABC$  be an equilateral triangle. Let  $M$  be a point on its circumcircle lying between  $B$  and  $C$ . Prove that  $MB + MC = MA$ .
3. (a) Define the power of a point with respect to a circle and describe, given a fixed circle, which point has the minimum possible power.  
(b) Let  $MA$  and  $MB$  be the two tangents from a point  $M$  outside a circle to the circle. Let also  $C, D$  be the points where a third line through  $M$  intersects the same circle. Show that  $(AC)(BD) = (AD)(BC)$ .
4. (a) Define the radical axis of two circles.  
(b) Show that if two circles are not intersecting, then their radical axis does not intersect either of them.
5. (a) Define the radical center of three circles.  
(b) Prove that the radical axes of three circles whose centers are not collinear, taken two at a time, are concurrent.