EXAM 3 - MATH 325

Thursday, March 27, 2003
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Read each problem very carefully before starting to solve it. Each question is worth 10 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) If two lines through a point P meet a circle at points A, B and C, D, respectively, then $PA \cdot PB = PC \cdot PD$.

   (b) If circles are constructed on two cevians as diameters, their radical axis passes through the orthocenter H of the triangle.

2. (a) State Simson's Theorem and then define the *Simson line* of a point with respect to a triangle.

   (b) If the diagonals of a quadrilateral inscribed in a circle are perpendicular, then the distance of the center from one of the sides is equal to one-half the length of the opposite side.

3. (a) State Ptolemy's Theorem.

   (b) Prove *Nagel's Theorem*: The radius of a circumcircle of ABC through A is perpendicular to the line joining the feet of the altitudes through B and C.

4. (a) Prove the *Ptolemy's Third Theorem*: Let $ABCD$ be a quadrilateral inscribed in a circle with $AB = \alpha, BC = \beta, CD = \gamma, AD = \delta, AC = \mu$ and $BD = \lambda$. Show that $\frac{\lambda}{\mu} = \frac{\alpha\beta+\gamma\delta}{\alpha\delta+\beta\gamma}$.

   (b) Use Ptolemy's Theorem and Ptolemy's Third Theorem to compute the lengths of the diagonals of a quadrilateral inscribed in a circle in terms of the lengths of its four sides. (In other words compute λ and μ above in terms of α, β, γ and δ).

5. (a) State the Butterfly Theorem.

   (b) State Morley's Theorem.