EXAM 4 - MATH 325
Thursday, April 17, 2003
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 10 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) State Pierre Varignon's Theorem and give the definition of Varignon parallelogram.

   (b) Show that, if one diagonal divides a quadrangle into two triangles of equal area, it bisects the other diagonal.

2. (a) Give Brahmagupta's formula for the area of a cyclic quadrangle and Heron's formula for the area of a triangle.

   (b) If a quadrangle with sides a, b, c, d is inscribed in one circle and circumscribed about another circle, its area K is given by $K^2 = abcd$.

3. (a) Use Heron's formula to find an expression for the length of the altitude of a triangle in terms of the lengths of its sides.

   (b) The sum of the squares of the sides of any quadrangle equals the sum of the squares of the diagonals plus four times the square of the segment joining the midpoints of the diagonals.

4. (a) Give the definition of the inner and the outer Napoleon triangles.

   (b) Let ABC be a triangle and O_2 the vertex of the outer Napoleon triangle corresponding to angle B and N_3 the vertex of the inner Napoleon triangle corresponding to angle C. Show that AN_3O_2 is similar to ABC.

5. (a) State Menelaus's Theorem.

   (b) Prove the *Theorem of Compagnon*: Divide the hypotenuse $a = BC$ of a right triangle ABC into three equal segments $BD = DE = EC$ and draw the line segments AD, AE. Prove that $AD^2 + AE^2 + DE^2 = \frac{2a^2}{3}$.