HOMEWORK 3 - MATH 325 DUE DATE: Tuesday, February 18

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. GOOD LUCK!!

1. If a Cevian AQ of an equilateral triangle ABC is extended to meet the circumcircle at P, then

$$\frac{1}{PB} + \frac{1}{PC} = \frac{1}{PQ}.$$

- 2. If lines PB and PD, outside a parallelogram ABCD, make equal angles with the sides BC and DC, respectively, as in Figure 1.9D, page 26, then $C\hat{P}B = D\hat{P}A$.
- 3. What is the algebraically smallest possible value that the power of a point can have with respect to a circle of given radius R? Which point has this critical power?
- 4. If PT and PU are tangents from P to two concentric circles, with T on the smaller, and if the segment PT meets the larger circle at Q, then $PT^2 PU^2 = QT^2$.
- 5. When the distance between the centers of two circles is greater than the sum of the radii, the circles have four common tangents. The midpoints of these four line segments are collinear.
- 6. Let PAB, AQB, ABR, P'BA, BQ'A, BAR' be six similar triangles all on the same side of their common side AB. (Three of them are shown in Figure 2.2D, page 34; the rest can be derived by reflection in the perpendicular bisector of the segment AB.) Those vertices of the triangles that do not lie on AB (namely, P, Q, R, P', Q', R') all lie on one circle. (*Hint*: Compare the powers of A and B with respect to the circle PQR.)